International journal of Engineering Research-Online
A Peer Reviewed International Journal
Articles available onlne http://www.ijoer.in

Vol.1., Issue.2., 2013

RESEARCH ARTICLE

ISSN: 2321-7758

# OBSERVATIONS ON RHOMBIC DODECAHEDRAL NUMBER

**M.A.GOPALAN [1],V.SANGEETHA [*],MANJU SOMANATH [2]**

[1]Professor,Department of Mathematics,Srimathi Indira Gandhi College,Trichy-2,India.

[*]Assistant Professor,Dept.of Mathematics, National College,Trichy-1,India.

Phone:9790451555,e-mail:prasansangee@gmail.com

[2] Assistant Professor,Dept.of Mathematics, National College,Trichy-1,India.

Article Received: 16/09/2013 | Article Revised on: 20/09/2013 | Article Accepted on:21/09/2013

V.SANGEETHA
Author for
Correspondence :

Email:prasansangee@
gmail.com

**ABSTRACT**

We obtain different relations among Rhombic Dodecahedral Number and other two, three and four dimensional figurate numbers.

**Keywords-**Polygonal number, Pyramidal number, Centered Polygonal number, Centered Pyramidal number, Rhombic Dodecahedral number, Special numbers.

**MSC Classification Number:** 11D99

## INTRODUCTION

Fascinated by beautiful and intriguing number patterns, famous mathematicians, share their insights and discoveries with each other and with readers. Throughout history, number and numbers [2,3,7-18] have had a tremendous influence on our culture and on our language. Polygonal numbers can be illustrated by polygonal designs on a plane. The - polygonal numbers can be summed up to form solid three dimensional figurate numbers called Pyramidal numbers [1,4,5and 6].In this communication,we deal with a special number called Rhombic Dodecahedral Number $RD(n) = 4n^3 - 6n^2 + 4n - 1$ and various interesting relations among these numbers are exhibited by means of theorems involving the relations.

**Notation**

$RD(n)$- Rhombic Dodecahedral number

$t_{m,n}$ - Polygonal number of rank n with sides m

$cp_{m,n}$- Centered Polygonal number of rank n with sides m

$P_n^m$- Pyramidal number of rank n with sides m

$CP_n^m$- Centered Pyramidal number of rank n with sides m

$PCS_m^n$ - Prism number of rank n with sides m

$P(n)$- Pronic number of rank n

$G(n)$- Gnomonic number

$O(n)$- Octahedral number

$CO(n)$- Centered Octahedral number

$I(n)$- Icosahedral number

$CI(n)$- Centered Icosahedral number

$D(n)$- Dodecahedral number

$SO(n)$- Stella octangula number

$CD(n)$- Centered Dodecahedral number

$CC(n)$- Centered Cube number

$CSq(n)$- Centered Square number

$TTH(n)$- Truncated Tetrahedral number

$TOH(n)$-Truncated Octahedral number

$PTP(n)$-Pentatope number

$HRD(n)$- Hauy Rhombic Dodecahedral number

$HO(n)$- Hauy Octahedral number

$N_d(n)$- $n^{th}$ d-dimensional nexus number

$g_m(n)$- m-gram number of rank n

$S_n$- Star number

**INTERESTING RELATIONS**

1. $RD(n) = 3P_n^{11} - 15t_{3,n-1} - CP_n^3 - 1$

**Proof:** $2RD(n) + 2CP_n^3 = 8n^3 - 12n^2 + 8n - 2 + n^3 + n$

$= 9n^3 - 12n^2 + 9n - 2$

$= 9n^3 + 3n^2 - 15n^2 - 6n + 15n - 2$

$= 6P_n^{11} - 15n^2 + 15n - 2$

$= 6P_n^{11} - 30t_{3,n-1} - 2$

$RD(n) = 3P_n^{11} - 15t_{3,n-1} - CP_n^3 - 1$

2. $RD(n) = 6P_n^6 - 14t_{3,n-1} - Csq(n)$

**Proof:** $RD(n) + Csq(n) = 4n^3 - 6n^2 + 4n - 1 + 2n^2 + 2n + 1$

$= 4n^3 - 4n^2 + 6n$

$= 4n^3 + 3n^2 - 7n^2 - n - 1 + 7n$

$= 4n^3 + 3n^2 - n - 7n^2 + 7n$

$= 6P_n^6 - 7n(n-1)$

$= 6P_n^6 - 14t_{3,n-1}$

$RD(n) = 6P_n^6 - 14t_{3,n-1} - Csq(n)$

3. $RD(n) = 3P_n^{10} - t_{17,n} - 1$

**Proof:** $RD(n) - 3P_n^{10} = 4n^3 - 6n^2 + 4n - 1 - \left(\frac{8n^3+3n^2-5n}{2}\right)$

$$= \frac{8n^3-12n^2+8n-2-8n^3-3n^2+5n}{2}$$

$$= \frac{-15n^2+13n-2}{2}$$

$$= \frac{-n(15n-13)}{2} - 1$$

$$= -t_{17,n} - 1$$

$$RD(n) = 3P_n^{10} - t_{17,n} - 1$$

4. $RD(n) = CC(n) + 6P_n^4 - 12t_{4,n} - 2$

**Proof:** $RD(n) - CC(n) = 4n^3 - 6n^2 + 4n - 1 - (2n^3 + 3n^2 + 3n + 1)$

$$= 2n^3 - 9n^2 + n - 2$$

$$= 2n^3 + 3n^2 - 12n^2 + n - 2$$

$$= 6P_n^4 - 12t_{4,n} - 2$$

$$RD(n) = CC(n) + 6P_n^4 - 12t_{4,n} - 2$$

5. $RD(n) = 3CP_n^8 - t_{14,n} - 1$

**Proof:** $RD(n) + t_{14,n} = 4n^3 - 6n^2 + 4n - 1 + 6n^2 - 5n$

$$= 4n^3 - n - 1$$

$$= n(4n^2 - 1) - 1$$

$$= 3CP_n^8 - 1$$

$$RD(n) = 3CP_n^8 - t_{14,n} - 1$$

6. $4RD(n) = TOH(n) + t_{20,n} + 2$

**Proof:** $4RD(n) - t_{20,n} - 2 = 4(4n^3 - 6n^2 + 4n - 1) - n(9n - 8) - 2$

$$= 16n^3 - 24n^2 + 16n - 4 - 9n^2 + 8n - 2$$

$$= 16n^3 - 33n^2 + 24n - 6$$

$$= TOH(n)$$

$$4RD(n) = TOH(n) + t_{20,n} + 2$$

7. $RD(n) = 2PCS_4^n - 4t_{3,n-1} - 1$

**Proof:** $RD(n) - 2PCS_4^n = 4n^3 - 6n^2 + 4n - 1 - (4n^3 - 4n^2 + 2n)$

$$= -2n^2 + 2n - 1$$

$$= -2n(n-1) - 1$$

$$= -4t_{3,n-1} - 1$$

$$RD(n) = 2PCS_4^n - 4t_{3,n-1} - 1$$

8. $RD(n) = 3HO(n) - 2G(n)$

**Proof:** $RD(n) + 2G(n) = 4n^3 - 6n^2 + 4n - 1 + 2(2n - 1)$

$$= 4n^3 - 6n^2 + 8n - 3$$

$$= 3HO(n)$$

$$RD(n) = 3HO(n) - 2G(n)$$

9. $RD(n) = P_n^3 + TTH(n) - 4t_{3,n-1} - 1$

**Proof:** $RD(n) - P_n^3 = 4n^3 - 6n^2 + 4n - 1 - \left(\frac{n^3+3n^2+2n}{6}\right)$

$$= \frac{24n^3-36n^2+24n-6-n^3-3n^2-2n}{6}$$

$$= \frac{23n^3-39n^2+22n-6}{6}$$
$$= \frac{23n^3-27n^2-12n^2+10n+12n-6}{6}$$
$$= \frac{23n^3-27n^2+10n}{6} - \frac{12n(n-1)}{6} - 1$$
$$= TTH(n) - 4t_{3,n-1} - 1$$
$$RD(n) = P_n^3 + TTH(n) - 4t_{3,n-1} - 1$$

10. $3RD(n) = HRD(n) - 3CP_n^8 + t_{38,n} + 4$

**Proof:** $3RD(n) + 3CP_n^8 = 12n^3 - 18n^2 + 12n - 3 + 4n^3 - n$
$$= 16n^3 - 18n^2 + 11n - 3$$
$$= 16n^3 - 36n^2 + 18n^2 + 28n - 17n - 7 + 4$$
$$= 16n^3 - 36n^2 + 28n - 7 + 18n^2 - 17n + 4$$
$$= HRD(n) + n(18n - 17) + 4$$
$$= HRD(n) + t_{38,n} + 4$$
$$3RD(n) = HRD(n) - 3CP_n^8 + t_{38,n} + 4$$

11. $RD(n) + P(n) + CP_n^6 - 2PCS_5^n \equiv -1 (mod\ 3)$

**Proof:** $RD(n) + P(n) + CP_n^6 = 4n^3 - 6n^2 + 4n - 1 + n(n+1) + n^3$
$$= 5n^3 - 6n^2 + 4n - 1 + n^2 + n$$
$$= 5n^3 - 5n^2 + 5n - 1$$
$$= 5n^3 - 5n^2 + 2n + 3n - 1$$
$$= PCS_5^n + 3n - 1$$
$$RD(n) + P(n) + CP_n^6 - 2PCS_5^n \equiv -1 (mod\ 3)$$

12. $RD(n) = 4CP_n^6 - 2t_{8,n} - 1$

**Proof:** $RD(n) = 4n^3 - 6n^2 + 4n - 1$
$$= 4n^3 - 2n(3n-2) - 1$$
$$= 4CP_n^6 - 2t_{8,n} - 1$$
$$RD(n) = 4CP_n^6 - 2t_{8,n} - 1$$

13. $RD(n) = CD(n) - 6P_n^8 - cp_{12,n} - 12t_{4,n} - 1$

**Proof:** $RD(n) + 6P_n^8 = 4n^3 - 6n^2 + 4n - 1 + (6n^3 + 3n^2 - 3n)$
$$= 10n^3 - 3n^2 + n - 1$$
$$= 10n^3 + 15n^2 - 18n^2 + 7n - 6n - 2 + 1$$
$$= 10n^3 + 15n^2 + 7n + 1 - 18n^2 - 6n - 2$$
$$= CD(n) - 6n^2 - 12n^2 - 6n - 1 - 1$$
$$= CD(n) - [6n(n+1) + 1] - 12n^2 - 1$$
$$= CD(n) - cp_{12,n} - 12t_{4,n} - 1$$
$$RD(n) = CD(n) - 6P_n^8 - cp_{12,n} - 12t_{4,n} - 1$$

14. $RD(n) = 24PTP(n) - n.CP_n^6 + G(n) - 6P_n^3 - 11t_{4,n}$

**Proof:** $RD(n) + n.CP_n^6 - G(n) = 4n^3 - 6n^2 + 4n - 1 + n^4 - 2n + 1$
$$= n^4 + 5n^3 - n^3 + 8n^2 - 14n^2 + 4n - 2n$$
$$= n^4 + 5n^3 + 8n^2 + 4n - n^3 - 14n^2 - 2n$$
$$= 24PTP(n) - (n^3 + 3n^2 + 2n) - 11n^2$$
$$= 24PTP(n) - 6P_n^3 - 11t_{4,n}$$
$$RD(n) = 24PTP(n) - n.CP_n^6 + G(n) - 6P_n^3 - 11t_{4,n}$$

15. $RD(n) = CI(n) + O(n) - 11t_{4,n} - 2$

**Proof:** $RD(n) - O(n) = 4n^3 - 6n^2 + 4n - 1 - \left(\frac{n(2n^2+1)}{3}\right)$

$$= \frac{12n^3 - 18n^2 + 12n - 3 - 2n^3 - n}{3}$$

$$= \frac{10n^3 + 15n^2 - 33n^2 + 11n + 3 - 6}{3}$$

$$= \frac{10n^3 + 15n^2 + 11n + 3}{3} - 11n^2 - 2$$

$$= CI(n) - 11t_{4,n} - 2$$

$$RD(n) = PTP(n) - n.CP_n^6 + G(n) - 6P_n^3 - 11t_{4,n}$$

16. $RD(n) + cp_{20,n} - 3CP_n^8 + t_{26,n} - 2G(n) - 2$ is a Square Integer.

**Proof:** $RD(n) + cp_{20,n} - 3CP_n^8 + t_{26,n} - 2G(n) = 16n^2 + 2$

$$= (4n)^2 + 2$$

$$RD(n) + cp_{20,n} - 3CP_n^8 + t_{26,n} - 2G(n) - 2 = (4n)^2$$

17. $RD(n) - 3CP_n^8 + G(n) + g_6(n) \equiv 0 \ (mod\ 1)$

**Proof:** $RD(n) - 3CP_n^8 + G(n) = 4n^3 - 6n^2 + 4n - 1 - 4n^3 + n + 2n - 1$

$$= -6n^2 + 5n - 1 + 2n - 1$$

$$= -6n^2 + 6n - n - 1 + 2n - 1$$

$$= -(6n^2 - 6n + 1) - n + 2n - 1$$

$$= -g_6(n) + n - 1$$

$$RD(n) - 3CP_n^8 + G(n) + g_6(n) = n - 1$$

$$RD(n) - 3CP_n^8 + G(n) + g_6(n) \equiv -1 \ (mod\ n)$$

18. $RD(n) = 2CP_n^{12} - g_6(n)$

**Proof:** $RD(n) + g_6(n) = 4n^3 - 6n^2 + 4n - 1 + 6n^2 - 6n + 1$

$$= 4n^3 - 2n$$

$$= 2n(2n^2 - 1)$$

$$= 2CP_n^{12}$$

$$RD(n) = 2CP_n^{12} - g_6(n)$$

19. $RD(n) + t_{18,n} + cp_{8,n} - G(n) - 3CP_n^8 - 1$ is a Nasty Number.

**Proof:** $RD(n) + t_{18,n} + cp_{8,n} - G(n) = 4n^3 - 6n^2 + 4n - 1 + 8n^2 - 7n$

$$+4n(n+1) + 1 - (2n - 1)$$

$$= 4n^3 + 6n^2 - n + 1$$

$$= n(4n^2 - 1) + 6n^2 + 1$$

$$= 3CP_n^8 + 6n^2 + 1$$

$$RD(n) + t_{18,n} + cp_{8,n} - G(n) - 3CP_n^8 - 1 = 6n^2.$$

20. $RD(n) + S_n + n - 2CP_n^9$ is a Cubic integer.

**Proof:** $RD(n) + S_n + n - 2CP_n^9 = 4n^3 - 6n^2 + 4n - 1 + 6n^2 - 6n + 1$

$$+n - n(3n^2 - 1)$$

$$= 4n^3 - n - 3n^3 + n$$

$$RD(n) + S_n + n - 2CP_n^9 = n^3.$$

21. $RD(n) + 120PTP(n) - N_4(n) - 38CP_n^3 + 2$ is a Nasty number.

**Proof:** $RD(n) + 120PTP(n) - N_4(n) = 4n^3 - 6n^2 + 4n - 1 + 5n^4 + 25n^3$

$$+40n^2 + 20n - 5n^4 - 10n^3$$

$$-10n^2 - 5n - 1$$

$$= 19n^3 + 24n^2 + 19n - 2$$
$$= 19n(n^2+1) + 24n^2 - 2$$
$$= 38CP_n^3 + 24n^2 - 2$$
$$RD(n) + 120PTP(n) - N_4(n) - 38CP_n^3 + 2 = 24n^2$$

22. $RD(n) + 3\,CO(n) + t_{8,n} - 6P_n^{10} \equiv 1(mod\ 2)$

**Proof:** $RD(n) + 3CO(n) + t_{8,n} = 4n^3 - 6n^2 + 4n - 1 + 4n^3 + 6n^2$
$$+8n + 3 + 3n^2 - 2n$$
$$= 8n^3 + 3n^2 + 10n + 2$$
$$= 8n^3 + 3n^2 + 15n - 5n + 2$$
$$= 6P_n^{10} + 15n + 2$$
$$RD(n) + 3\,CO(n) + t_{8,n} - 6P_n^{10} = 15n + 2$$
$$RD(n) + 3\,CO(n) + t_{8,n} - 6P_n^{10} \equiv 1(mod\ 2)$$

REFERENCES

[1]. Beiler A.H., “Ch.18 in Recreations in the Theory of Numbers”, The Queen of Mathematics Entertains, New York,Dover,(1996),184-199.

[2]. Bert Miller, “Nasty Numbers”, The Mathematics Teachers, 73(9), (1989),649.

[3]. Bhatia B.L., and Mohanty Supriya, “Nasty Numbers and their characterization”,Mathematical Education,I(1) (1985) 34-37.

[4]. Bhanu Murthy T.S.,Ancient Indian Mathematics, New Age International Publishers Limited,New Delhi, 1990.

[5]. Conway J.H and Guy R.K., The Book of Numbers,New York Springer-Verlag,(1996),44-48

[6]. Dickson L.E., The Book of Numbers,Chelsea Publishing Company, New York, 1952.

[7]. Horadam A.F., “Jacobsthal Representation Numbers”,Fib Quart.,34 (1996) 40-54.

[8]. Meyyappan M.,Ramanujam Numbers,S.Chand and Company Ltd.,First Edition,(1996)

[9]. Shailesh Shirali, “Primer on number Sequences”,Mathematics Student, 45 (2) (1997), 63-73.

[10]. Gopalan M.A. and Devibala S., “On Lanekel Number” Antarctica J.Math.,4(1) (2007),35-39.

[11]. Gopalan M.A.,Manju Somanath and Vanitha.N., “A Ramarkable Lanekel Sequence”,Proc.Nat.Acad.Sci. India 77 (A) II (2007),139-142

[12]. Gopalan M.A.,Manju Somanath and Vanitha.N., “On R2 Numbers”,Acta Ciencia Indica,XXXIIIM (2) (2007),617-619.

[13]. Gopalan M.A. and Gnanam A., “Star Numbers” Math.Sci.Res.J.,12 (12) (2008),303-308.

[14]. Gopalan M.A and Gnanam A., “A Notable Integer Sequence”Math. Sci.Res.J., 1(1) (2008) 7-15.

[15]. Gopalan M.A. and Gnanam A., “Magna Numbers” Indian Journal of Mathematical Sciences,5(1), (2009) 33-34.

[16]. Gopalan M.A. and Gnanam A., “Four dimensional Pyramidal numbers”Pacific Asian Journal of Mathematics,4(1) (2010),53-62.

[17]. Gopalan M.A., Manju Somanath and Geetha K., “Observations on Icosagonal Number” IJCER 3(5) (2013),28-34.

[18]. Gopalan M.A., Manju Somanath and Geetha K., “Observations on Icosagonal Pyramidal Number” IRJES 2(7) (2013),32-37.