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INTEGRAL SOLUTIONS OF SEXTIC NON-HOMOGENEOUS EQUATION WITH FIVE UNKNOWNS $(x^3+y^3)=z^3+w^3+6(x+y)t^5$

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ABSTRACT

The sextic non-homogeneous equation with five unknowns represented by the diophantine equation

$$(x^3+y^3)=z^3+w^3+6(x+y)t^5$$

is analyzed for its patterns of non-zero distinct integral solutions. A few interesting relations between the solutions and special numbers are exhibited.

KEYWORDS
Integral solutions,sextic non-homogeneous equation ,lattice points.
M.Sc 2000 mathematics subject classification: 11D41

NOTATIONS

$t_{m,n}$: Polygonal number of rank n with size m

P_n^m : Pyramidal number of rank n with size m

S_n : Star number of rank n

So_n : Stella octangular number of rank n

$\Pr_n$: Pronic number of rank n

$Ct_{m,n}$: Centered Polygonal number of rank n with size m

INTRODUCTION

The theory of diophantine equations offers a rich variety of fascinating problems [1-4]. Particularly, in [5,6], sextic equations with 3 unknowns are studied for their integral solutions. [7-11] analyse sextic equations with 4 unknowns for their non-zero integer solutions. This communication analyses a sextic equation with 5 unknowns given by $(x^3+y^3)=z^3+w^3+6(x+y)t^5$.

Infinitely many non-zero integer quintuples (x,y,z,w,t) satisfying the above equation are obtained. Various interesting properties among the values of x,y,z,w and t are presented.

METHOD OF ANALYSIS

The diophantine equation representing a non-homogeneous sextic equation is

$(x^3+y^3)=z^3+w^3+6(x+y)t^5$ (1)

Introducing the linear transformations

$x=u+v, y=u-v$, $z=u+p$,

$w=u-p$ (2)

in (1), it leads to

$v^2=p^2+2t^5$ (3)

It is observed that (3) has infinitely many integral solutions. For simplicity and clear understanding, we exhibit below the different choices for v and p satisfying (3). when (i) t is even (ii) t is a perfect square and (iii) t is 2(a square).Knowing the vales of v,p,t and using (2) we get infinitely many non-zero integral solutions of (1).

2:1 Let $t=2k$ (4)

Using (4) in (3), we have

$v^2-p^2=2^6k^5$ (5)

which may be considered to be equivalent to system of double equations in different ways.

Illustration: 1

Write $v+p=16k^4$, $v-p=4k$

On solving, the values of v and p are given by $v=8k^4+2k$, $p=8k^4-2k$

Substituting these values in (2), the corresponding integral solutions of (1) are presented by

$x=x(u,k)=u+8k^4+2k$

$y=y(u,k)=u-8k^4-2k$

$z=z(u,k)=u+8k^4-2k$

$w=w(u,k)=u-8k^4+2k$

$t=t(k)=2k$

Properties

1. Each of the following is a nasty integer.

(a) $6(x(u,k)-y(u,k)+64P_k^5-\mathrm{Pr}_{4k})$

(b) $x(u,k)-w(u,k)-3{t_{6,k}}^2-3P_k^6+3t_{10,k}-6\,\mathrm{Pr}_k+6t_{4,k}$

2. $z(u,k)-w(u,k)-4{t_{6,k}}^2-32P_k^5+4t_{12,k}\equiv 0(\mathrm{mod}\,12)$

3. $x(u,k)-z(u,k)+4\,\mathrm{Pr}_k-t_{10,k}\equiv 0(\mathrm{mod}\,11)$

4. $z(u,k)-y(u,k)-So_k+4P_k^5-2\,\mathrm{Pr}_k-t_{6,k}+2t_{4,k}$ is a biquadratical integer.

Illustration: 2

Write $v+p=16k^3, v-p=4k^2$

On solving, the values of v and p are given by $v=8k^3+2k^2$, $p=8k^3-2k^2$

Substituting these values in (2), the corresponding integral solutions of (1) are given by

$x = x(u,k) = u + 8k^3 + 2k^2$

$y = y(u,k) = u - 8k^3 - 2k^2$

$z = z(u,k) = u + 8k^3 - 2k^2$

$w = w(u,k) = u - 8k^3 + 2k^2$

$t = t(k) = 2k$

Properties

1. $x(u,k) - w(u,k) - 16P_k^5 - 32t_{4,k}$ is a nasty number.

2. Each of the following is a cubical integer.

(a) $x(u,k) - y(u,k) - 4So_k - 4\Pr_k$

(b) $z(u,k) - w(u,k) - 3t_{6,k}{}^2 - 3P_k^6 + 3t_{10,k} - 6\Pr_k + 6t_{4,k}$

3. $w(u,k) - y(u,k) - S_k - 4\Pr_k - Ct_{4,k} \equiv 0 \pmod 2$

4. $x(u,k) - z(u,k) - 4t_{4,k} + 4\Pr_k - Ct_{8,k} = -1$

5. $x(u,k)z(u,k)$ is the difference of two squares.

2:2 Take $t = \alpha^2$ (6)

Substituting (6) in (3), we get

$v^2 = p^2 + 2(\alpha^5)^2$ (7)

The assumption $v = m + n, p = m = n$ (7a)

in (7)leads to $4mn = 2(\alpha^5)^2$ (8)

which is satisfied by $m = 2^{19}n^9, \alpha = 4n$ (9)

using (9) in (7a),we have

$v = 2^{19}n^9 + n$, $p = 2^{19}n^9 - n$ (10)

Substituting the values of v, p and α in (2) and (6), the corresponding integral solutions of (1) are obtained as follows

$x = x(u,n) = u + 2^{19}n^9 + n$

$y = y(u,n) = u - 2^{19}n^9 - n$

$z = z(u,n) = u + 2^{19}n^9 - n$

$w = w(u,n) = u - 2^{19}n^9 + n$

$t = t(n) = 16n^2$

Further note that (7) is also satisfied by

$\alpha^5 = 2rs, v = 2r^2 + s^2, p = 2r^2 - s^2$ (11)

Choose r and s such that

$rs = 2^4\beta^5$ (11a)

Now, choose $r = 2^3\beta^3, s = 2\beta^2$ (12)

and thus $\alpha = 2\beta$ (13)

using (12),(13) in (11) and (2),

$v = 2(2^6\beta^6) + 4\beta^4$ (14)

The corresponding integral solutions of (1) are obtained as follows

$x = x(u,\beta) = u + 2^7\beta^6 + 4\beta^4$

$y = y(u,\beta) = u - 2^7\beta^6 - 4\beta^4$

$z = z(u,\beta) = u + 2^7\beta^6 - 4\beta^4$

$w = w(u,\beta) = u - 2^7\beta^6 + 4\beta^4$

$t = t(\beta) = 4\beta^2$

It is worth to mention here that there are other choices for r and s in (11a).A similar analysis may be performed to obtain the other patterns of solutions to (1).

2:3 Considering $t = 2E^2$ (15)

in (3),it can be written as

$v^2 = p^2 + (8E^5)^2$ (16)

Note that (16) is similar to the well known pythogorean equation .Using the most citied solution of the standard pythogorean, we have

$p = 2mn, v = m^2 + n^2$ (17)

$8E^5 = m^2 - n^2$ (17a)

Write $m + n = 4E^3, m - n = 2E^2$

On solving, the values of m and n are given by

$m = 2E^3 + E^2, n = 2E^3 - E^2$

Substituting these values in (17) and (2), the corresponding integral solutions of (1) are given by

$$x = x(u, E) = u + 2(4E^6 + E^4)$$
$$y = y(u, E) = u - 2(4E^6 + E^4)$$
$$z = z(u, E) = u + 2(4E^6 - E^4)$$
$$w = w(u, E) = u - 2(4E^6 - E^4)$$
$$t = t(E) = 2E^2$$

It is to be noted that (17a) may be factorized in a different way, namely, $m + n = 8E^3, m - n = E^2$, the corresponding solutions to (1) are obtained proceeding as above, and so on.

CONCLUSION

In a similar manner, one may consider the other choices for the system of double equations for obtaining the v and p values in each of the above three cases (i) to (iii) and obtain respectively the corresponding integral solutions.

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