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RESEARCH ARTICLE

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ON THE NON-HOMOGENEOUS HEPTIC EQUATION WITH FOUR UNKNOWNS

$$(x^2+y^2)(x+y)^4 = z^4w^3$$

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ABSTRACT

The non-homogeneous heptic equation with four unknowns represented by the diophantine equation $(x^2+y^2)(x+y)^4 = z^4w^3$ is analyzed for its patterns of non-zero distinct integral solutions. Various interesting relations between the solutions and special numbers, namely, Pyramidal numbers, Pronic numbers, Centered Hexagonal Pyramidal numbers, fourth dimensional figurate numbers are exhibited.

KEYWORDS: Non-homogeneous heptic equation ,Pyramidal numbers, Pronic numbers, fourth dimensional figurate numbers

M.Sc 2000 mathematics subject classification: 11D41

NOTATIONS

$t_{m,n}$: Polygonal number of rank n with size m

P_n^m : Pyramidal number of rank n with size m

Pr_n : Pronic number of rank n

j_n : Jacobsthal-Lucas number of rank n

J_n : Jacobsthal number of rank n

$Cf_{3,n,6}$: Centered Hexogonal Pyramidal number of rank n

$Cf_{4,n,6}$: Fourth dimensional Hexogonal figurate number of rank n

INTRODUCTION

The theory of diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous and non-homogeneous equations of higher degree have aroused the interest of numerous Mathematicians since antiquity [1-3].Particularly in [4,5] special equations of sixth degree with four and five unknowns are studied. In [6-9] heptic equations with three,four and five unknowns are analysed. This communication analyses a hextic equation with four unknowns given by $(x^2+y^2)(x+y)^4 = z^4 w^3$. Infinitely many non-zero integer quadtuples (x, y, z, w) satisfying the above equation are obtained. Various interesting properties among the values of x, y, z and w are presented.

METHOD OF ANALYSIS

The diophantine equation representing a non-homogeneous heptic equation is

$$(x^2+y^2)(x+y)^4 = z^4 w^3 \quad (1)$$

Introducing the linear transformations

$$x = u+v,\ y = u-v,\ z = 2^n u,\ n \ge 1 \quad (2)$$

in (1),it leads to

$$u^2+v^2 = 2^{4n-5} w^3 \quad ,\ n \ge 1 \quad (3)$$

Assume $w = w(a,b) = a^2+b^2,\ a,b \ne 0$ (4)

and write 2^{4n-5} as

$$2^{4n-5} = (2^{2n-3}+i2^{2n-3})(2^{2n-3}-i2^{2n-3}) \quad (5)$$

Substituting (5),(4) in (3), and employing the method of factorization, define

$$u+iv = 2^{2n-3}(1+i)\left(a^3-3ab^2+i(3a^2b-b^3)\right) \quad (6)$$

Equating real and imaginary parts in (6),we get

$$\left.\begin{aligned} u &= 2^{2n-3}(a^3-3ab^2-3a^2b+b^3) \\ v &= 2^{2n-3}(a^3-3ab^2+3a^2b-b^3) \end{aligned}\right\} \ldots \quad (7)$$

Substituting (7) in (2), the corresponding non-zero integral solutions of (1) are represented by

$$\left.\begin{aligned} x &= x(a,b) = 2^{2n-2}(a^3-3ab^2) \\ y &= y(a,b) = 2^{2n-2}(b^3-3a^2b) \\ z &= z(a,b) = 2^{3n-3}(a^3-3ab^2-3a^2b+b^3) \\ w &= w(a,b) = (a^2+b^2) \end{aligned}\right\} \ldots\ldots \quad (8)$$

Properties:

1. Each of the following is a cubic integer

(a) $$\frac{x(a,b)+3a(2^{2n-2})w(a,b)}{4^n}$$

(b) $$2\left[z(a,1)+3(2^{3n})w(a,1)-2^{3n}(Cf_{3,a,6}-6t_{3,a}+3t_{4,a})\right]$$

2. Each of the following is a nasty number

(a) $$6\left[4\left[x(a,1)+y(a,1)\right]-(j_{2n}-1)Cf_{3,a,6}+\left[9J_{2n}+3\right)Pr_a\right]$$

(b) $$3\left[y(2^n,2^n)\right]$$

3. $$2z(a,b) = \left[x(a,b)+y(a,b)\right](j_n-(-1)^n)$$

4. $$4\frac{x\left(a(a+1),1\right)}{(j_{2n}-1)}+6t_{3,a} = Cf_{3,a+1,6}(2P_a^5-t_{4,a})$$

5. $$4\frac{y\left(\,,a(a+1)\right)}{(3J_{2n}+1)}+2P_a^5+3Pr_a = 6Cf_{4,a,6}$$

A few numerical examples are given in the table below

n	a	b	x	y	z	w
2	2	1	8	-44	-72	5
3	2	1	32	-176	-576	5
4	2	1	128	-704	-4608	5
5	3	2	-2304	-11776	-225280	13
6	4	2	16384	-90112	-2359296	20

However, we have an another pattern that is illustrated below:

Rewrite (3) as

$$2(u^2+v^2) = 2^{4n-4} w^3 \quad (9)$$

Also, consider 2 and 2^{4n-4} as

$$\left.\begin{aligned} 2 &= (1+i)(1-i) \\ 2^{4n-4} &= (1+i)^{4n-4}(1-i)^{4n-4} \end{aligned}\right\} \ldots\ldots \quad (10)$$

Using (10),(4) in (9) and employing the method of factorization, define

$$u - v = 4^{n-1}\left[\cos\pi(n-1)(a^3 - 3a^2b) - \sin\pi(n-1)(3a^2b - b^3)\right]$$

$$u + v = 4^{n-1}\left[\cos\pi(n-1)(3a^2b - b^3) + \sin\pi(n-1)(a^3 - 3ab^2)\right]$$

Solving the above system of equations and using (2),we get

$$x = 2^{2n-2}\left[\cos\pi(n-1)(3a^2b - b^3) + \sin\pi(n-1)(a^3 - 3ab^2)\right]$$
$$y = 2^{2n-2}\left[\cos\pi(n-1)(a^3 - 3ab^2) + \sin\pi(n-1)(b^3 - 3a^2b)\right]$$
$$z = 2^{3n-3}\left[\cos\pi(n-1)(a^3 - 3a^2b + 3a^2b - b^3) + \sin\pi(n-1)(a^3 - 3ab^2 - 3a^2b + b^3)\right]$$

The above equation and (4) represent the non-zero integral solutions to (1)

Remark

It is worth to mention here that ,when $n = 1$ in (1),it simplifies to $2(u^2 + v^2) = w^3$.Observe that 2 can be factorized as product of two gaussian integers in three different ways , namely

$$2 = (1+i)(1-i) \qquad (11)$$
$$= \frac{(7+i)(7-i)}{25} \qquad (12)$$
$$= \frac{(41+i)(41-i)}{841} \qquad (13)$$

The integral values of x, y, z satisfying (1)by taking (11) are obtained by satisfying $n = 1$
in (8).following the analysis presented above ,the integral solutions of (1) for the choices (12) and (13) are exhibited below:

Integral solutions for choice (13)

$$x = 600A^3 - 800B^3 - 1800AB^2 + 2400A^2B$$
$$y = 800A^3 + 600B^3 - 2400AB^2 - 1800A^2B$$
$$z = 1400A^3 - 4200AB^2 + 600A^2B - 200B^3$$
$$w = 100A^2 + 100B^2$$

Integral solutions for choice (14)

$$\begin{bmatrix} x = (1682)^2(1160A^3 - 3480AB^2 + 3654A^2B - 1218B^3) \\ y = (1682)^2(1218A^3 - 3654AB^2 - 3480A^2B + 1160B^3) \\ z = (1682)^2(2378A^3 - 7134AB^2 + 174A^2B - 58B^3) \\ w = (1682)^2(A^2 + B^2) \end{bmatrix}$$

Conclusion

To conclude one may search for other pattern of solutions and their corresponding properties.

REFERENCES

[1]. L.E.Dickson, History of Theory of Numbers, Vol.11, Chelsea Publishing company, New York (1952).

[2]. L.J.Mordell, Diophantine equations, Academic Press, London(1969)

[3]. Carmichael ,R.D.,The theory of numbers and Diophantine Analysis,Dover Publications, New York (1959)

[4]. M.A.Gopalan, S. Vidhyalakshmi and K.Lakshmi, On the non-homogeneous sextic equation $x^4 + 2(x^2 + w)x^2y^2 + y^4 = z^4$, IJAMA, 4(2), 171-173 (2012)

[5]. M.A.Gopalan,S.Vidhyalakshmi and K.Lakshmi, Integral Solutions of the sextic equation with five unknowns $x^3 + y^3 = z^3 + w^3 + 3(x+y)T^5$, IJESRT, 502-504, Dec.2012

[6]. M.A.Gopalan and sangeetha. G, parametric integral solutions of the heptic equation with 5 unknowns $x^4 - y^4 + 2(x^3 + y^3)(x - y) = 2(X^2 - Y^2)z^5$,Bessel J. Math,1(1), 17-22, 2011.

[7]. M.A.Gopalan and sangeetha.G, On the heptic diophantine equations with 5 unknowns $x^4 - y^4 = (X^2 - Y^2)z^5$,Antarctica J.Math, 9(5), 371-375, 2012.

[8]. Manjusomnath, G.sangeetha and M.A.Gopalan, On the non-homogeneous heptic equations with 3 unknowns

$x^3+(2^p-1)y^5=z^7$, Diophantine J.math 1(2), 117-121, 2012

[9]. M.A.Gopalan,G.Sumathi and S.Vidhyalakshmi, On the non-homogeneous heptic equations with four unknowns $xy(x+y)+zw^6$, IJESRT ,2(5), 1313-1317 May 2013.