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# INTEGRAL SOLUTION OF $43x^2 + y^2 = z^2$

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**ABSTRACT**

The ternary quadratic homogeneous equation representing homogeneous cone given by $43x^2 + y^2 = z^2$ is analyzed for its non-zero distinct integer points on it. Three different patterns of integer points satisfying the cone under consideration are obtained. A few interesting relations between the solutions and special number patterns namely Polygonal number , Pyramidal number , Octahedral number, Pronic number ,Stella octangular number , Pentatope number and Nasty number are presented. Also knowing an integer solution satisfying the given cone , three triples of integers generated from the given solution are exhibited.

**Keywords:** Ternary homogeneous quadratic, integral solutions

**2010 Mathematics Subject Classification: 11D09**

**Notations:**

$P_n^m$ - Pyramidal number of rank n with size m.

$T_{m,n}$ - Polygonal number of rank n with size m.

$\Pr_n$ - Pronic number of rank n

## INTRODUCTION

The ternary quadratic Diophantine equations offer an unlimited field for research due to their variety [1, 21]. For an extensive review of various problems, one may refer [2-20]. This communication concerns with yet another interesting ternary quadratic equation $43x^2 + y^2 = z^2$ representing a cone for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

**METHOD OF ANALYSIS:**

The ternary quadratic equation under consideration is $43x^2 + y^2 = z^2$ (1)

To start with it is seen that the triples $(k, 21k, 22k), (2k+1, 2k^2+2k-21, 2k^2+2k+22)$ and $(2rs, r^2-43s^2, r^2+43s^2)$ Satisfy (1).

However, we have other choices of solutions to (1) which are illustrated below:
consider (1) as

$$43x^2 + y^2 = z^2 * 1 \quad (2)$$

Assume $z = a^2 + 43b^2$ (3)

Write 1 as

$$1 = \frac{((21+2n-2n^2)+i(2n-1)\sqrt{43})((21+2n-2n^2)-i(2n-1)\sqrt{43})}{(22-2n+2n^2)^2} \quad (4)$$

Substituting (3) and (4) in (2) and employing the method of factorization. define

$$y + i\sqrt{43}x = (a + i\sqrt{43}b)^2 \frac{[(21+2n-2n^2)+i(2n-1)\sqrt{43}]}{(22-2n+2n^2)^2}$$

Equating the real and imaginary parts in the above equation, we get

$$x = \frac{[2(21+2n-2n^2)ab-(a^2-43b^2)(2n-1)]}{22-2n+2n^2}$$

$$y = \frac{[(21+2n-2n^2)(a^2-43b^2)-86ab(2n-1)]}{22-2n+2n^2}$$

Replacing a by $(22-2n+2n^2)A$, b by $(22-2n+2n^2)B$ in the above equation corresponding integer solutions to (1) are given by

$$\left.\begin{array}{l} x = (22-2n+2n^2)[(A^2-43B^2)(2n-1)+\{2AB(21+2n-2n^2)\}] \\ y = (22-2n+2n^2)[(A^2-43B^2)(21+2n-2n^2)-\{86AB(2n-1)\}] \\ z = (22-2n+2n^2)^2(A^2+43B^2) \end{array}\right\} \quad (A)$$

For simplicity and clear understanding, taking n=1 in (A), the corresponding integer solutions of (1) are given by

$$x = 22A^2 - 946B^2 + 924AB$$

$$y = 462A^2 - 19866B^2 - 1892AB$$

$$z = 22^2(A^2 + 43B^2)$$

**Properties:**

$$x(A,1) - t_{(46,A)} \equiv -1 \pmod{945}$$

$$x(A,1) - 2t_{(24,A)} \equiv -2 \pmod{944}$$

$$x(n+1, n^2) - t_{(46,n)} + 946t_{(4,n^2)} - 1848p_n^5 \equiv 22 \pmod{65}$$

$$x(n+1, n) - t_{(1850,n)} - 924pr_n \equiv 22 \pmod{879}$$

$$y(A,1)-t_{(926,A)}\equiv -1263(\mathrm{mod}1431)$$

$$y(n+1,n^2)-t_{(926,n)}+19866t_{(4,n^2)}+3784p_n^5\equiv 462(\mathrm{mod}1385)$$

$$y(n+1,n)-t_{(38810,n)}+1892pr_n\equiv 462(\mathrm{mod}18479)$$

Each of the following represents a nasty number

$$66z(A,A)=6(22A)^2$$

$$y(14A,2A)=6(2A)^2$$

It is worth to note that 1 in (2) may also be represented as

$$1=\frac{\{(43-4n^2)+i(4n)\sqrt{43}\}\{(43-4n^2)-i(4n)\sqrt{43}\}}{(43+4n^2)^2}$$

Following the analysis presented above, the corresponding integer solutions to (1) are found to be

$$\left.\begin{aligned} x&=(43+4n^2)[(A^2-43B^2)(4n)+\{2AB(43-2n^2)\}]\\ y&=(43+4n^2)[(A^2-43B^2)(43-4n^2)-\{344nAB\}]\\ z&=(43+4n^2)^2(A^2+43B^2)\end{aligned}\right\}\qquad \text{(B)}$$

For the sake of simplicity, taking n=1 in (B) ,the corresponding integer solution of (1) are given by

$$x=188A^2-8084B^2+3666AB$$

$$y=1833A^2-78819B^2-16168AB$$

$$z=47^2(A^2+43B^2)$$

**Properties:**

$$x(A,1)-t_{(378,A)}\equiv -376(\mathrm{mod}3853)$$

$$x(A,1)-2t_{(190,A)}\equiv -380(\mathrm{mod}3854)$$

$$x(n+1,n^2)-t_{(378,n)}+8084t_{(4,n^2)}-7332p_n^5\equiv 188(\mathrm{mod}563)$$

$$x(n+1,n)-t_{(15794,n)}-3666pr_n\equiv 188(\mathrm{mod}7519)$$

$$y(A,1)-t_{(3668,A)}\equiv -7139(\mathrm{mod}14336)$$

$$y(n+1,n^2)-t_{(3668,n)}+78819t_{(4,n^2)}+32336p_n^5\equiv 1833(\mathrm{mod}5498)$$

$$y(n+1,n)-t_{(153974,n)}+16168pr_n\equiv 1833(\mathrm{mod}73319)$$

**Generation of integer solutions**

Let $(x_0,y_0,z_0)$ be any given integer solution of (1)

Then, each of the following triples of integers satisfies (1):

**Triple 1** : $(x_{n1},y_{n1},z_{n1})$

$$x_{n1}=44x_0+2y_0-7z_0$$

$$y_{n1}=86x_0+5y_0-14z_0$$

$$z_{n1}=301x_0+14y_0-48z_0$$

**Triple 2** : $(x_{n2},y_{n2},z_{n2})$

$$x_{n2}=\frac{1}{42}[\{43(-21)^n-(21)^n\}x_0+\{(21)^n-(-21)^n\}z_0]$$

$$y_{n2}=21^n y_0$$

$$z_{n2}=\frac{1}{42}[\{43(-21)^n-43(21)^n\}x_0+\{43(21)^n-(-21)^n\}z_0]$$

**Triple 3:** $(x_{n3},y_{n3},z_{n3})$

$$x_{n3}=\frac{1}{44}[\{43(-22)^n+(22)^n\}x_0+\{(-22)^n-(22)^n\}y_0]$$

$$y_{n3}=\frac{1}{44}[\{43(-22)^n-43(22)^n\}x_0+\{43(22)^n+(-22)^n\}y_0]$$

$$z_{n3}=22^n z_0$$

**Triple 4:** $(x_{n4},y_{n4},z_{n4})$

$$x_{n4}=4^n x_0$$

$$y_{n4}=\frac{1}{8}[\{9(4)^n-(-4)^n\}y_0+\{3(-4)^n-3(4)^n\}z_0]$$

$$z_{n4}=\frac{1}{8}[\{3(4)^n-3(-4)^n\}y_0+\{9(-4)^n-(4)^n\}z_0]$$

**CONCLUSION**

In this paper, we have presented three different patterns of non-zero distinct integer solutions of the homogeneous cone given by $43x^2+y^2=z^2$.To conclude, one may search for other patterns of solution and their corresponding properties.

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