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RESEARCH ARTICLE

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ON THE TERNARY CUBIC EQUATION $4(X^2+Y^2)-7XY=24Z^3$

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ABSTRACT

The ternary cubic Diophantine equation $4(X^2+Y^2)-7XY=24Z^3$ is analyzed for its infinitely many non-zero distinct integral solutions. A few interesting properties among the solutions are presented.

Keywords: Ternary cubic, Integral solutions.

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INTRODUCTION

Searching for the integral solutions to the cubic (homogeneous or non-homogeneous) Diophantine equations is an interesting concept as it can be seen from [1, 2, 3]. In [4-20] a few special cases of ternary cubic Diophantine equations are studied. In this communication, we present the integral solutions of yet another ternary cubic equation $4(X^2+Y^2)-7XY=24Z^3$. A few interesting relations between the solutions and special numbers are obtained.

Notations:

$T_{m,n}$ = Polygonal number of rank n with size m.

P_n^m = Pyramidal number of rank n with size m.

$\Pr_n$ = Pronic number of rank n.

2. Method of Analysis:

The cubic equation under consideration is

$$4(X^2+Y^2)-7XY=24Z^3. \qquad (1)$$

Assuming $X=u+v, Y=u-v, u\neq v$ (2)

in (1), it is written as

$$u^2 + 15v^2 = 24z^3 \tag{3}$$

Here, we present three different choices of solutions of (3) and hence, obtain three different patterns of solutions to (1).

write $Z = Z(a,b) = a^2 + 15b^2,\ a,b \neq 0$ (4)

2.1 PATTERN: I

Write, 24 as

$$24 = (3 + i\sqrt{15})(3 - i\sqrt{15}) \tag{5}$$

Substituting (4) and (5) in (3) and employing the method of factorization, define

$$(u + i\sqrt{15}v) = (3 + i\sqrt{15})(a + i\sqrt{15}b)^3$$

Equating real and imaginary parts, we have

$$\left.\begin{aligned} u &= 3a^3 - 45a^2b - 135ab^2 + 225b^3 \\ v &= a^3 + 9a^2b - 45ab^2 - 45b^3 \end{aligned}\right\} \tag{6}$$

Substituting the above values in (2), the corresponding values of x and y are

$$\left.\begin{aligned} X &= X(a,b) = 4a^3 - 36a^2b - 180ab^2 + 180b^3 \\ Y &= Y(a,b) = 2a^3 - 54a^2b - 90ab^2 + 270b^3 \end{aligned}\right\} \tag{7}$$

Thus (4) and (7) represent the non-zero distinct integer solutions to (1).

Properties:

- $X(a,1) - 8P_a^5 + T_{82,a} \equiv 180 \pmod{219}$
- $Y(a,1) - 4P_a^5 + T_{114,a} \equiv 270 \pmod{145}$
- $X(a,1) - Y(a,1) - 4P_a^5 - T_{82,a} + T_{114,a} \equiv 16 \pmod{74}$
- $X(a,a+1) - 8P_a^5 + 544T_{4,a} - 432P_a^5 \equiv 0 \pmod{2}$
- $Z(a,a+1) - 16T_{4,a} \equiv 0 \pmod{3}$

2.2 PATTERN: II

(3) can be written as

$$u^2 + 15v^2 = 24z^3 * 1 \tag{8}$$

write 1 as,

$$1 = \frac{(7 + i\sqrt{15})(7 - i\sqrt{15})}{64} \tag{9}$$

Substituting (4) and (9) in (8) and employing the method of factorization, define

$$(u + i\sqrt{15}v) = (3 + i\sqrt{15})(a + i\sqrt{15}b)^3 \frac{(7 + i\sqrt{15})}{8}$$

Equating real and imaginary parts, we have

$$\left.\begin{aligned} u &= \frac{1}{8}[6a^3 - 450a^2b - 270ab^2 + 2250b^3] \\ v &= \frac{1}{8}[10a^3 + 18a^2b - 450ab^2 - 90b^3] \end{aligned}\right\} \tag{10}$$

As our interest is on finding integer solutions we choose a and b suitably so that the values of u and v are integers.

Replacing a by 8A and b by 8B in (10) and (4), the corresponding values of u, v and z are

$$u = 384A^3 - 28800A^2B - 17280AB^2 + 144000B^3$$
$$v = 640A^3 + 1152A^2B - 28800AB^2 - 5760B^3$$
$$Z = Z(A,B) = 64A^2 + 960B^2 \quad \textbf{(11)}$$

Substituting the above values of u and v in (2), we get

$$\left.\begin{array}{l} x(A,B) = 1024A^3 - 27648A^2B - 46080AB^2 + 138240B^3 \\ y(A,B) = -256A^3 - 29952A^2B + 11520AB^2 + 149760B^3 \end{array}\right\} \quad (12)$$

Thus, (11) and (12) represent the non-zero distinct integer solutions to (1).

Properties:

- $X(A,1) - 2048P_A^5 + 28672T_{4,A} \equiv 138240 \pmod{46080}$
- $Y(A,1) + 512P_A^5 + 29696T_{4,A} \equiv 149760 \pmod{11520}$
- $Z(A,A+1) - 1024T_{4,A} \equiv 0 \pmod 2$
- $X(A,A+1) - 131072P_A^5 - 229376T_{4,A} \equiv 138240 \pmod{414720}$
- $X(A,1) - Y(A,1) - 2560P_A^5 - 1024T_{4,A} \equiv 11520 \pmod{57600}$

2.3 PATTERN: III

Instead of (5), write 24 as

$$24 = \frac{(9 + i\sqrt{15})(9 - i\sqrt{15})}{4} \quad (13)$$

Substituting (4) and (13) in (3) and employing the method of factorization, define

$$(u + i\sqrt{15}v) = \frac{(9 + i\sqrt{15})}{2}(a + i\sqrt{15}b)^3$$

Equating real and imaginary parts, we have

$$\left.\begin{array}{l} u = \frac{1}{2}[9a^3 - 45a^2b - 105ab^2 + 225b^3] \\ v = \frac{1}{2}[a^3 + 27a^2b - 45ab^2 - 135b^3] \end{array}\right\} \quad (14)$$

As our interest is on finding integer solutions we choose a and b suitably so that the values of u and v are integers.

Replacing a by 2A and b by 2B in (14) and (4), the corresponding values of u, v and z are

$$u = 36A^3 - 180A^2B - 1620AB^2 + 900B^3$$
$$v = 4A^3 + 108A^2B - 180AB^2 - 540B^3$$
$$Z = Z(A,B) = 4A^2 + 60B^2 \quad (15)$$

Substituting the above values of u and v in (2), we get

$$\left.\begin{array}{l} x(A,B) = 40A^3 - 72A^2B - 1800AB^2 + 360B^3 \\ y(A,B) = 32A^3 - 288A^2B - 1440AB^2 + 1440B^3 \end{array}\right\} \quad (16)$$

Thus, (15) and (16) represent the non-zero distinct integer solutions to (1).

Properties:

- $X(A,1) - 80P_A^5 + T_{226,A} \equiv 360 \pmod{111}$

- $Y(A,1) - 64P_A^5 + T_{642,A} \equiv 1440 \pmod{319}$
- $X(A, A+1) + 1432P_A^5 + 2632T_{4,A} \equiv 0 \pmod 2$
- $Z(A, A+1) - 68T_{4,A} \equiv 0 \pmod 2$
- $X(A, A+1) - Z(A, A+1) + 1432P_A^5 + 1364T_{4,A} \equiv 530 \pmod{970}$

3. **CONCLUSION:**

It is worth to mention here**,** that, using (4), (9) and (13) in (8), a different choice of integer solutions to (1) is obtained. To conclude, one may search for other patterns of solutions to (1) along with their properties.

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