International journal of Engineering Research-Online
A Peer Reviewed International Journal
Articles available online http://www.ijoer.in

Vol.2., Issue.1., 2014

RESEARCH ARTICLE

ISSN:2321-7758

# ON THE TERNARY QUADRATIC DIOPHANTINE EQUATION

$$29x^2 + y^2 = z^2$$

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Article Received: 04/03/2014 | Article Revised on: 12/03/2014 | Article Accepted on:13/03/2014

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**ABSTRACT**

The ternary quadratic homogeneous equation representing homogeneous cone given by $29x^2 + y^2 = z^2$ is analyzed for its non-zero distinct integer points on it. Three different patterns of integer points satisfying the cone under consideration are obtained. A few interesting relations between the solutions and special number patterns namely Polygonal number , Pyramidal number, Octahedral number, Pronic number and Nasty number are presented. Also knowing an integer solution satisfying the given cone , three triples of integers generated from the given solution are exhibited.

**Keywords:** Ternary homogeneous quadratic, integral solutions

**2010 Mathematics Subject Classification: 11D09**

## INTRODUCTION

The ternary quadratic Diophantine equations offer an unlimited field for research due to their variety [1, 21]. For an extensive review of various problems, one may refer [2-20]. This communication concerns with yet another interesting ternary quadratic equation $29x^2 + y^2 = z^2$ representing a cone for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

**NOTATIONS:**

$P_n^m$ - Pyramidal number of rank n with size m.

$T_{m,n}$ - Polygonal number of rank n with size m.

$\Pr_n$ - Pronic number of rank n

$OH_n$ - Octahedral number of rank n

**METHOD OF ANALYSIS**

The ternary quadratic equation under consideration is

$$29x^2 + y^2 = z^2 \tag{1}$$

To start with it is seen that the triples $(k,14k,15k),(2k+1,2k^2+2k-14,2k^2+2k+15)$ and $(2rs, r^2-29s^2, r^2+29s^2)$ satisfy (1).

However, we have other choices of solutions to (1) which are illustrated below:

Consider (1) as

$$29x^2 + y^2 = z^2 * 1 \tag{2}$$

Assume $z = a^2 + 29b^2$ (3)

Write 1 as

$$1 = \frac{[(14+2n-2n^2)+i(2n-1)\sqrt{29}][(14+2n-2n^2)-i(2n-1)\sqrt{29}]}{(15-2n+2n^2)^2} \tag{4}$$

Substituting (3) and (4) in (2) and employing the method of factorization, define

$$y + i\sqrt{29}x = \frac{(a^2+29b^2)[(14+2n-2n^2)+i(2n-1)\sqrt{29}]}{(15-2n+2n^2)^2}$$

Equating the real and imaginary parts in the above equation, we get

$$x = \frac{[2(14+2n-2n^2)ab-(a^2-29b^2)(2n-1)]}{15-2n+2n^2}$$

$$y = \frac{[(14+2n-2n^2)(a^2-43b^2)-28ab(2n-1)]}{15-2n+2n^2}$$

Replacing a by $(15-2n+2n^2)A$, b by $(15-2n+2n^2)B$ in the above equation corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= (15-2n+2n^2)[(A^2-29B^2)(2n-1)+\{2AB(14+2n-2n^2)\}] \\ y &= (15-2n+2n^2)[(A^2-29B^2)(14+2n-2n^2)-\{58AB(2n-1)\}] \\ z &= (15-2n+2n^2)^2(A^2+29B^2) \end{aligned}\right\} \tag{A}$$

For simplicity and clear understanding, taking n=1 in (A), the corresponding integer solutions of (1) are given by

$$x = 15A^2 - 435B^2 + 420AB$$

$$y = 210A^2 - 6090B^2 - 870AB$$

$$z = 15^2(A^2 + 29B^2)$$

**Properties:**

- $x(A,1) - T_{32,A} \equiv -1 \pmod{434}$
- $x(A,1) - T_{16,A} - T_{18,A} \equiv -2 \pmod{433}$
- $x(n+1,n^2) - T_{32,n} + 435T_{4,n^2} - 840p_n^5 \equiv 15 \pmod{44}$
- $x(n+1,n) - T_{842,n} - 420pr_n \equiv 15 \pmod{389}$

- $y(A,1)-x(A,1)-T_{392,A)} \equiv -175 (\bmod 1096)$
- $y(A,1)-z(A,1)+T_{32,A} \equiv -239 (\bmod 884)$
- $z(A,1)+T_{452,A} \equiv 29 (\bmod 224)$

Each of the following represents a nasty number

- $10[z(A,A)-y(A,A)]=6(150A)^2$
- $-5y(A,A)=6(75A)^2$

It is worth to note that 1 in (2) may also be represented as

$$1=\frac{\{(29-4n^2)+i((4n)\sqrt{29})\}\{(29-4n^2)-i((4n)\sqrt{29})\}}{(29+4n^2)^2}$$

Following the analysis presented above, the corresponding integer solutions to (1) are found to be

$$\left.\begin{aligned} x&=(29+4n^2)[(A^2-29B^2)(4n)+\{2AB(29-4n^2)\}] \\ y&=(29+4n^2)[(A^2-29B^2)(29-4n^2)-\{232nAB\}] \\ z&=(29+4n^2)^2(A^2+29B^2) \end{aligned}\right\} \quad \text{(B)}$$

For the sake of simplicity, taking n=1 in (B) ,the corresponding integer solution of (1) are given by

$$x=132A^2-3828B^2+1650AB$$
$$y=825A^2-23925B^2-7656AB$$
$$z=33^2(A^2+43B^2)$$

**Properties:**

- $x(A,1)-T_{266,A} \equiv -266 (\bmod 1781)$
- $x(A,2A^2+1)-49500OH_A+15312T_{4,A^2}+T_{30362,A} \equiv -3828 (\bmod 15179)$
- $x(n+1,n^2)-T_{266,n}+3828T_{4,n^2}-3300p_n^5 \equiv 132 (\bmod 395)$
- $x(n+1,n)-T_{7394,n}-1650pr_n \equiv 132 (\bmod 3959)$
- $x(1,B)-y(1,B)-T_{40196,B} \equiv -693 (\bmod 29402)$
- $z(n(n+1),n+2)+T_{60986,n}-1089T_{4,n^2}-4356p_n^5 \equiv 30491 (\bmod 95833)$
- $z(1,B)-y(1,B)-T_{111014,B} \equiv 264 (\bmod 63161)$

**Generation of integer solutions:**

Let $(x_0,y_0,z_0)$ be any given integer solution of (1)

Then, each of the following triples of integers satisfies (1):

**Triple 1** : $(x_{n1},y_{n1},z_{n1})$

$$x_{n1}=4^n x_0$$

$$y_{n1}=\frac{1}{8}[\{9(4)^n-(-4)^n\}y_0+\{3(-4)^n-3(4)^n\}z_0]$$

$$z_{n1}=\frac{1}{8}[\{3(4)^n-3(-4)^n\}y_0+\{9(-4)^n-(4)^n\}z_0]$$

**Triple 2** : $(x_{n2}, y_{n2}, z_{n2})$

$$x_{n2} = \frac{1}{28}[\{29(-14)^n - (14)^n\}x_0 + \{(14)^n - (-14)^n\}z_0]$$

$$y_{n2} = 14^n y_0$$

$$z_{n2} = \frac{1}{28}[\{29(-14)^n - 29(14)^n\}x_0 + \{29(14)^n - (-14)^n\}z_0]$$

**Triple 3:** $(x_{n3}, y_{n3}, z_{n3})$

$$x_{n3} = \frac{1}{30}[\{29(-15)^n + (15)^n\}x_0 + \{(-15)^n - (15)^n\}y_0]$$

$$y_{n3} = \frac{1}{30}[\{29(-15)^n - 29(15)^n\}x_0 + \{29(15)^n + (-15)^n\}y_0]$$

$$z_{n3} = 15^n z_0$$

**CONCLUSION**

In this paper, we have presented three different patterns of non-zero distinct integer solutions of the homogeneous cone given by $29x^2 + y^2 = z^2$.To conclude, one may search for other patterns of solution and their corresponding properties.

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