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RESEARCH ARTICLE

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OBSERVATIONS ON ICOSAHEDRAL NUMBER

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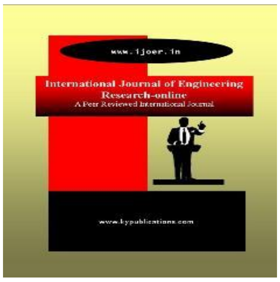

ABSTRACT

We obtain different relations among Icosahedral number and other two, three and four dimensional figurate numbers.

INTRODUCTION

The numbers that can be represented by a regular arrangement of points are called the polygonal numbers (also known as two dimensional figurate numbers). The polygonal number series can be summed to form solid three dimensional figurate numbers called Pyramidal numbers that be illustrated by pyramids[1].Numbers have varieties of patterns[2-18] and varieties of range and richness. In this communication we deal with Icosahedral numbers given by $I_n = \frac{n\,(5n^2 - 5n + 2)}{2}$ and various interesting relations among these numbers are exhibited by means of theorems involving the relations.

Keywords: Polygonal numbers, Pyramidal numbers and Special numbers
2010 MSC Classification Code: 11D99

Notation

$t_{m,n}$ = Polygonal number of rank n with sides m

p_n^m = Pyramidal number of rank n with sides m

$F_{m,n,p}$ = m-dimensional figurate number of rank n where generated polygon is of p sides

ja_n = Jacobsthal number

jal_n = Jacobsthal lucas number

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$ct_{m,n}$ = Centered Polygonal number of rank n with sides m

g_n = Gnomonic number of rank n with sides m

p_n = Pronic number

$carl_n$ = Carol number

mer_n = Mersenne number, where n is prime

Tha_n = Thabit ibn kurrah number

INTERESTING RELATIONS

1) $2I_n + 3n = 6p_n^7 - t_{18,n}$

Proof:

$$2I_n = \; 5n^3 + 3n^2 - 2n \; - \; 8n^2 - 7n \; - 3n$$

$$= 6p_n^7 - t_{18,n} - 3n$$

2) $I_n = \frac{1}{2} \; 3cp_n^{10} - t_{12,n}$

Proof:

$$2I_n = \; 5n^2 - 2n \; - \; 5n^2 - 4n$$

$$= 3cp_n^{10} - t_{12,n}$$

3) $2I_n - n \; 2ct_{3,n} + ct_{4,n} \; + t_{22,n} + 10n = 0$

Proof:

$$2I_n = n \; 3n^2 + 3n + 2 \; + n \; 2n^2 + 2n + 1 \; - 10n^2 - n$$

$$= n \; 2ct_{3,n} + ct_{4,n} \; - t_{22,n} - 10n$$

4) $2^{1-n} I_{2^n} = 5ky_n - 15(2^n) + 7$

Proof:

$$\frac{2I_{2^n}}{2^n} = 5 \; 2^{2n} \; - 5 \; 2^n \; + 2$$

$$= 5 \; 2^{2n} + 2^{n+1} - 1 \; - 15(2^n) + 7$$

$$2^{1-n} I_{2^n} = 5ky_n - 15(2^n) + 7$$

5) The following represents a nasty number

i) $2^{1-n} I_{2^n} - jal_{2n} - ja_{2n} - 3carl_n - mer_n$

Proof:

$$\frac{2I_{2^n}}{2^n} = 5 \; 2^{2n} \; - 5 \; 2^n \; + 2$$

$$= \; 2^{2n} + 1 \; + \; 2^{2n} - 1 \; + 3 \; 2^{2n} - 2 * 2^n - 1 \; + 2^n + 5$$

$$2^{1-n} I_{2^n} - jal_{2n} - ja_{2n} - 3car1_n - mer_n = 6$$

ii) $2I_{2n+1} - 2p_n^{14} - 6p_n^5 - 33cp_n^6 - 17n - 2$

Proof:

$$2I_{2n+1} = 40n^3 + 40n^2 + 14n + 2$$

$$= 4n^3 + n^2 - 3n + 3\ n^3 + n^2 + 33n^3 + 36n^2 + 17n + 2$$

$$= 2p_n^{14} + 6p_n^5 + 33cp_n^6 + 17n + 2 + 36n^2$$

$$2I_{2n+1} - 2p_n^{14} - 6p_n^5 - 33cp_n^6 - 17n - 2 = 36n^2$$

iii) $I_n + 7n - 3cp_n^5 + t_{19,n}$

Proof:

$$2I_n = 5n^3 + n - 17n^2 + 15n + 12n^2 - 14n$$

$$= 3cp_n^5 - t_{19,n} + 6n^2 - 7n$$

$$I_n + 7n - 3cp_n^5 + t_{19,n} = 6n^2$$

6) $2I_{n+2} = 6cp_n^5 + 2ct_{3,n} + ct_{4,n} + 8g_n + 29$

Proof:

$$2I_{n+2} = 5n^3 + 25n^2 + 42n + 24$$

$$= 5n^3 + n + 23n^2 + 23n + 2 + 2n^2 + 2n + 1 + 16n + 29$$

$$= 6cp_n^5 + 2ct_{3,n} + ct_{4,n} + 8g_n + 29$$

7) $2\ I_{n+1} - I_{n-1} - 3ct_{20,n} + 25g_n + 15 = 0$

Proof:

$$2\ I_{n+1} - I_{n-1} = 30n^2 - 20n + 13$$

$$= 3\ 10n^2 + 10n + 1 - 25\ 2n - 1 - 15$$

$$= 3ct_{20,n} - 25g_n - 15$$

8) $I_{2n+1} - 40p_n^5 \equiv 1 \mod 7$

Proof:

$$2I_{2n+1} = 40n^3 + 40n^2 + 14n + 2$$

$$= 80p_n^5 + 14n + 2$$

$$I_{2n+1} - 40p_n^5 \equiv 1 \mod 7$$

9) $I_n + n = 3cp_n^5 - t_{7,n}$

Proof:

$$2I_n = 5n^3 + n - 5n^2 - 3n - 2n$$

$$= 6cp_n^5 - 2t_{7,n} - 2$$

$$I_n + n = 3cp_n^5 - t_{7,n}$$

10) $I_n + 9n = 3p_n^7 - t_{18,n} + 4t_{3,n}$

Proof:

$$I_n = 3p_n^7 - 4t_{3,n} + 6n \qquad (1)$$

$$2I_n = 6p_n^7 - t_{18,n} - 3n \qquad (2)$$

Subtract (2) and (1), we get

$$I_n + 9n = 3p_n^7 - t_{18,n} + 4t_{3,n}$$

11) $2\ I_n - p_{n+1}^5 - t_{3,n+1} - 3cp_n^8 + t_{12,n} \equiv -4 \mod 13$

Proof:

$$2I_n = n^3 + 4n^2 + 5n + 2 + n^3 + 3n + 2 + 4n^3 - 10n^2 - 6n - 4$$

$$= 2p_{n+1}^5 + 2t_{3,n+1} + 4n^3 - n - 10n^2 - 8n - 13n - 4$$

$$= 2p_{n+1}^5 + 2t_{3,n+1} + 6cp_n^8 - 2t_{12,n} - 13n - 4$$

$$2\ I_n - p_{n+1}^5 - t_{3,n+1} - 3cp_n^8 + t_{12,n} \equiv -4 \mod 13$$

12) $I_n = cp_n^{15} - 5t_{3,n} + 5n$

Proof:

$$2In = 5n^3 - 3n - 5\ n^2 + n + 10n$$

$$= 2cp_n^{15} - 10t_{3,n} + 10n$$

$$I_n = cp_n^{15} - 5t_{3,n} + 5n$$

13) $2^{1-n} I_n = ky_n + 4carl_n + Mer_n + 8$

Proof:

$$\frac{2I_{2^n}}{2} = 2^{2n} + 2 * 2^n - 1 + 4\ 2^{2n} - 2 * 2^n - 1 + 2^n - 1 + 8$$

$$= ky_n + 4carl_n + Mer_n + 8$$

14) $2In - 5cp_n^6 + s_n - p_n \equiv 1 \mod 5$

Proof:

$$2In = 5n^3 - 6n^2 - 6n + 1 + n^2 + n - 5n + 1$$

$$= 5cp_n^6 - s_n + p_n - 5n + 1$$

$$2In - 5cp_n^6 + s_n - p_n \equiv 1 \mod 5$$

15) $2I_n = 3\ cp_n^5 + p_n^7\ - t_{7,n} - 8t_{3,n} + 5n$

Proof:

$$I_n = 3cp_n^5 - t_{7,n} - n \tag{1}$$

$$I_n = 3p_n^7 - 8t_{3,n} + 6n \tag{2}$$

Add (1) and (2), we get

$$2I_n = 3\ cp_n^5 + p_n^7\ - t_{7,n} - 8t_{3,n} + 5n$$

16) $I_n + 2n = 3F_{4,m,4} + p_n^{11} - t_{11,n}$

Proof:

$$I_n = \frac{1}{2}\ \ 2n^3 + 3n^2 + n\ +\ 3n^3 + n^2 - 2n\ -\ 9n^2 - 7n\ - 4n$$

$$= \frac{1}{2}\ 6F_{4,m,4} + 2p_n^{11} - 2t_{11,n} - 4n$$

$$I_n + 2n = 3F_{4,m,4} + p_n^{11} - t_{11,n}$$

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