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RESEARCH ARTICLE

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AN INTERESTING TRANSCENDENTAL EQUATION WITH SIX UNKNOWNS

$$\sqrt[2]{x^2+y^2-xy}-\sqrt[3]{X^2+Y^2}=z^2-w^2$$

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ABSTRACT

The transcendental equation with six unknowns involving surds represented by the equation $\sqrt[2]{x^2+y^2-xy}-\sqrt[3]{X^2+Y^2}=z^2-w^2$ is analyzed for its patterns of non-zero distinct integral solutions. Infinitely many non-zero integer sextuple (x, y, X, Y, z, w) satisfying the above equation are obtained.

Three different patterns for finding the solution to the above problem are discussed. The relations between the solutions and the Polygonal numbers, Pyramidal numbers, Pronic number, Jacobsthal number, Jacobsthal-Lucas number, Octahedral number, kynea number, Centered pyramidal numbers and Four Dimensional Figurative numbers are presented.

KEYWORDS: Transcendental equation, integral solutions, the Polygonal numbers, Pyramidal numbers, Pronic number, Jacobsthal number, Jacobsthal-Lucas number, Octahedral number, kynea number, Centered pyramidal numbers and Four Dimensional Figurative numbers.

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NOTATIONS:

$T_{m,n}$ -Polygonal number of rank n with size m

P_n^m - Pyramidal number of rank n with size m

PR_n - Pronic number of rank n

OH_n - Octahedral number of rank n

SO_n -Stella octangular number of rank n

S_n -Star number of rank n

J_n -Jacobsthal number of rank of n

j_n - Jacobsthal-Lucas number of rank n

KY_n -kynea number of rank n

$CP_{n,3}$ - Centered Triangular pyramidal number of rank n

$CP_{n,6}$ - Centered hexagonal pyramidal number of rank n

$F_{4,n,3}$ - Four Dimensional Figurative number of rank n whose generating polygon is a triangle

$F_{4,n,5}$ - Four Dimensional Figurative number of rank n whose generating polygon is a pentagon.

INTRODUCTION

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems are algebraic equations. It seems that much work has not been done to obtain integral solutions of transcendental equations. In this context one may refer $1-16$. This communication analyses a transcendental equation with six unknown given by $\sqrt[2]{x^2+y^2-xy}-\sqrt[3]{X^2+Y^2}=z^2-w^2$. Infinitely many non-zero integer sextuples (x,y,X,Y,z,w) satisfying the above equation are obtained.

Method of Analysis:

The Diophantine equation representing the transcendental equation is given by

$$\sqrt[2]{x^2+y^2-xy}-\sqrt[3]{X^2+Y^2}=z^2-w^2 \tag{1}$$

To start with, it is observed that (1) is satisfied by the following 2 non-zero distinct integer Sextuples:

$$(x,y,X,Y,z,w):\ (\alpha^2-12k^2+4\alpha k, \alpha^2-12k^2-4\alpha k,\ \alpha^3+4k^2\alpha, 2k\alpha^2+8k^3, 3k, k),$$

$$(\alpha^2-12k^2+4\alpha k, \alpha^2-12k^2-4\alpha k,\ \alpha^3+4k^2\alpha, 2k\alpha^2+8k^3, 1+2k^2, 1-2k^2)$$

However, we have other patterns of solutions which are illustrated below:

To start with, the transformations

$$\left.\begin{array}{l} x=p^2-3q^2+2pq, y=p^2-3q^2-2pq, \\ X=p(p^2+q^2), Y=q(p^2+q^2) \end{array}\right\} \tag{2}$$

in (1) leads to

$$2q^2=z^2-w^2 \tag{3}$$

Pattern1:

Let $z=\beta^2+2\gamma^2$ (4)

Rewriting (3) as

$$w^2+2q^2=z^2 \tag{5}$$

and using the method of factorisation, define

$$(w+i\sqrt{2}q)=(\beta+i\sqrt{2}\gamma)^2$$

Equating real and imaginary parts, we get

$$w=\beta^2-2\gamma^2, q=2\beta\gamma \tag{6}$$

In view of (2), (4) and (6), we have the integral solution of (1) as

$$\left.\begin{array}{l} x=\alpha^2-12\beta^2\gamma^2+4\alpha\beta\gamma \\ y=\alpha^2-12\beta^2\gamma^2-4\alpha\beta\gamma \\ X=\alpha(\alpha^2+4\beta^2\gamma^2) \\ Y=2\beta\gamma(\alpha^2+4\beta^2\gamma^2) \\ z=\beta^2+2\gamma^2 \\ w=\beta^2-2\gamma^2 \end{array}\right\} \tag{7}$$

Properties:

1. $3(\alpha+1)[x(a,a)+y(a,a)+4(z(a,a)+w(a,a))+2(X(a,a)+Y(a,a))-(8+4\alpha^2)T_{4,a}+8(\alpha-3)(PR_{a^2}-T_{4,a})+8(SO_a.CP_{a,6})+8T_{4,a}^2]$ is a nasty number.

2. $\alpha(\alpha+1)^3[x(a,a)+X(a,a)+z(a,a)-(8\alpha+3)T_{4,a}+12(2T_{3,a}.T_{4,a}-CP_{a,6})]$ is a cubic integer.

3. $x(a,a)-y(a,a)-w(a,a)-8\alpha(2P_a^5-CP_{a,6})-T_{4,a}=0$

4. $z(2a,a).w(2a,a)=12[24F_{4,a,3}-36P_a^3+14T_{3,a}+SO_a-2CP_{a,6}]$

5. $X(a,1).Y(a,1)-2\alpha^5(2T_{4,a}-T_{6,a})-16\alpha^3CP_{a,6}\equiv 0 \pmod{32}$

Pattern2:

Now, rewrite (3) as,

$$w^2+2q^2=z^2*1 \qquad (8)$$

Also 1 can be written as

$$1=\frac{((2-k^2)+i2\sqrt{2}k)((2-k^2)-i2\sqrt{2}k)}{(2+k^2)^2} \qquad (9)$$

Substituting (4) and (9) in (8) and using the method of factorisation, define

$$(w+i\sqrt{2}q)=\frac{(2-k^2+i2\sqrt{2}k)(\beta-i2\sqrt{2}\gamma)^2}{(2+k^2)} \qquad (10)$$

Equating real and imaginary parts in (10) we get

$$\left.\begin{aligned} w&=\frac{1}{2+k^2}[(2-k^2)(\beta^2-2\gamma^2)-8k\beta\gamma] \\ q&=\frac{1}{2+k^2}[2k(\beta^2-2\gamma^2)+2\beta\gamma(2-k^2)] \end{aligned}\right\} \qquad (11)$$

considering (11), (4) and (2) and performing some algebra, the corresponding values of x, y, X, Y, z, w are obtained as follows:

$$\left.\begin{aligned} x&=p^2-3q^2+2pq \\ y&=p^2-3q^2-2pq \\ X&=p(p^2+q^2) \\ Y&=q(p^2+q^2) \\ z&=(\beta^2+2\gamma^2)(2+k^2)^2 \\ w&=(2+k^2)[(\beta^2-2\gamma^2)(2-k^2)-8k\beta\gamma] \end{aligned}\right\} \qquad (12)$$

where $q=(2+k^2)[2k(\beta^2-2\gamma^2)+(2-k^2)2\beta\gamma], k=1,2,......$

Now, rewrite (3) as,

$$z^2-2q^2=w^2*1 \qquad (13)$$

Also 1 can be written as

$$1=\frac{(2+k^2+2\sqrt{2}k)(2+k^2-2\sqrt{2}k)}{(2-k^2)^2} \qquad (14)$$

Following the same procedure as above we get the integral solution of (1) as

$$\left.\begin{aligned} x &= p^2-3q^2+2pq \\ y &= p^2-3q^2-2pq \\ X &= p(p^2+q^2) \\ Y &= q(p^2+q^2) \\ z &= (2-k^2)[(2+k^2)(\beta^2+2\gamma^2)+8k\beta\gamma] \\ w &= (2-k^2)^2(\beta^2-2\gamma^2) \end{aligned}\right\} \qquad (15)$$

where

$$q=(2-k^2)[2k(\beta^2+2\gamma^2)+(2+k^2)2\beta\gamma], k=1,2,......$$

Pattern3:

Using the transformations

$$\left.\begin{aligned} &x=p^2-3q^2+2pq, y=p^2-3q^2-2pq, \\ &X=p(p^2+q^2), Y=q(p^2+q^2), \\ &z=r+s, w=r-s \end{aligned}\right\} \qquad (16)$$

in (1) we have

$$q^2=2rs \qquad (17)$$

The choices

$$r=2^{2n-1}\beta^{2n}, \quad s=\gamma^{2n}, \qquad \beta,\gamma=1,2,..... \qquad (18)$$

in (17),give

$$q=(2\beta\gamma)^n \qquad (19)$$

Using (16), (18) and (19) the integral solution of (1) is obtained as

$$\left.\begin{aligned} x &= \alpha^2-3(2\beta\gamma)^{2n}+2\alpha(2\beta\gamma)^n \\ y &= \alpha^2-3(2\beta\gamma)^{2n}-2\alpha(2\beta\gamma)^n \\ X &= \alpha(\alpha^2+(2\beta\gamma)^{2n}) \\ Y &= (2\beta\gamma)^n(\alpha^2+(2\beta\gamma)^{2n}) \\ z &= 2^{2n-1}\beta^{2n}+\gamma^{2n} \\ w &= 2^{2n-1}\beta^{2n}-\gamma^{2n} \end{aligned}\right\} \qquad (20)$$

Note;

By taking r *and* s differently, we get different values for z *and* w

Properties:

1. $z(a,a)-w(a,a)-4P^5_{a^n}+2CP_{a^n,6}=0$

2. $8\alpha^2[x(a,a)+y(a,a)+6(2^{2n}T_{4,a^{2n}})]$ is a biquadratic integer.

3. $z(a,a).w(a,a)-(2^{4n-2}-1)(6F_{4,a^n,5}-3CP_{a^n,6}-2T_{4,a^n})=0$

4. $3(\alpha+1)[x(a,a)+y(a,a)+2\{z(a,a)+w(a,a)+X(a,a)+Y(a,a)\}-(2^{2n+1}\alpha-4(2^{2n})-\alpha^2 2^{2n+1})T_{4,a^n}-\alpha^2 2^{n+1}PR_{a^n}-2^{3n+1}CP_{a^n,6}]$ is a nasty integer.

5. $X(a,1).Y(a,1)-\alpha(2^n)[\alpha^4(2T_{3,a^n}-T_{4,a^n})+2^{2n+1}\alpha^2 CP_{a^n,6}+2^{4n}(2CP_{a^n,3}.T_{4,a^n}-CP_{a^n,3})]=0$

CONCLUSION

In conclusion, one may search for different patterns of solutions to (1) and their corresponding properties.

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