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RESEARCH ARTICLE

CHARACTERIZATION THEOREMS ON THERMOSOLUTAL CONVECTION IN COUPLE-STRESS FLUID IN A POROUS MEDIUM

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ABSTRACT

Thermosolutal instability of Veronis type in couple-stress fluid in a porous medium is considered. Following the linearized stability theory and normal mode analysis, the paper mathematically established the condition for characterizing the oscillatory motions which may be neutral or unstable, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a couple-stress fluid of infinite horizontal extension and finite vertical depth in a porous medium, are necessarily non-oscillatory, in the regime

$$R_s \leq 4\pi^4\left\{\frac{F}{P_l} + \frac{1}{E^{/} p_3 \varepsilon}\right\}$$

Where R_s is the Thermosolutal Rayliegh number, p_3 is the thermosolutal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the couple stress parameter. The result is important since it hold for all wave numbers and for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid. A similar characterization theorem is also proved for Stern type of configuration.

Key Words: Thermal convection; Couple-Stress Fluid; PES; Rayleigh number; thermosolutal Rayleigh number

MSC 2000 No.: 76A05, 76E06, 76E15; 76E07; 76U05.

INTRODUCTION

The theoretical and experimental results on thermal convection in a fluid layer, in the absence and presence of rotation and magnetic field have been given by Chandrasekhar [1]. The buoyancy force can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [2]. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. The migration of moisture in fibrous insulation, bio/chemical contaminants transport in environment, underground disposal of nuclear wastes, magmas, groundwater, high quality crystal production and production of pure medication are some examples where double-diffusive convection is involved. Examples of particular interest are provided by ponds built to trap solar heat Tabor and Matz [2] and some Antarctic lakes Shirtcliffe[4]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The study of thermosolutal convection in fluid saturated porous media has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust Lister[5]. Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice - versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context Mc Donnel [6]. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Also the magnetic field in double-diffusive convection has its importance in the fields of engineering, for example, MHD generators and astrophysics particularly in explaining the properties of large stars with a helium rich core. Stommel and Fedorov [7] and Linden [8] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [9] explained a double - diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason et al. [10] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge.

The theory of couple-stress fluid has been formulated by Stokes [11]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze - film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded – bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is clear or yellowish. According to the theory of Stokes [11], couple-stresses appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [12] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is due to its content of the hyaluronic acid, a fluid of high viscosity, near to gel. Goel et al. [13] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [14] have considered a couple - stress fluid with suspended particles heated from below. In another study, Sunil et al. [15] have considered a couple- stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al. [16] have considered the thermal instability of a layer of couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects.

Pellow and Southwell [17] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [18] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee[19] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al [20]. However no such result existed for non-Newtonian fluid configurations in general and in particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal [20] have characterized the oscillatory motions in couple-stress fluid.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the thermosolutal convection in couple-stress fluid in porous medium, this article attempts to study the couple-stress fluid convection of Veronis and Stern type configuration in a porous medium, and it has been established that the onset of instability in a Rivlin-Ericksen viscoelastic fluid heated from below in a porous medium of Veronis [2] type configuration, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F satisfy the inequality

$$R_s \le 4\pi^4\left\{\frac{F}{P_l}+\frac{1}{E^{/}p_3\varepsilon}\right\}$$

for all wave numbers and for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid. A similar characterization theorem is also proved for Stern [22] type of configuration.

FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_0, ρ_0 and C_0 and at the upper surface z = d are T_d, ρ_d and C_d respectively, and that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta^{/}$ $\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The gravity field $\vec{g}(0,0,-g)$, pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability k_1.

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Let p, ρ, T, C, α, $\alpha^{/}$, g, and $\vec{q}(u,v,w)$ denote respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of couple-stress fluid (Chandrasekhar [1]; Stokes [11]) are

$$\frac{1}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q}, \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$E\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\kappa\nabla^2 T, \quad (3)$$

$$E^{/}\frac{\partial C}{\partial t}+\left(\vec{q}.\nabla\right)C=\kappa^{/}\nabla^2 C, \quad (4)$$

$$\rho = \rho_0\,[1 - \alpha\,(T-T_0) + \alpha^{/}\,(C-C_0)], \quad (5)$$

Where the suffix zero refers to values at the reference level z = 0 and in writing equation (1), use has been made of Boussinesq approximation. Here E = $\in+(1-\in)\left(\frac{\rho_s C_s}{\rho_0 C_i}\right)$ is a constant and $E^{/}$ is a constant analogous to E but corresponding to solute rather that heat; ρ_s, C_s and ρ_o, C_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The kinematic viscosity ν, couple-stress viscosity $\mu^{/}$, the thermal diffusivity κ and the solute diffusivity $\kappa^{/}$ are all assumed to be constants.

The steady state solution is

$$\vec{q}(u,v,w)=(0,0,0),\ T = T_0-\beta z,\ C = C_0 - \beta^{/}z,$$

$$\rho = \rho_0\,(1+ \alpha\beta z - \alpha^{/}\beta^{/}z). \quad (6)$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution, and let δp, δρ, θ, γ and $\vec{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C and velocity $\vec{q}(0,0,0)$. The change in density δρ, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = -\rho_0\,(\alpha\theta - \alpha^{/}\gamma). \quad (7)$$

Then the linearized perturbation equations become

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\vec{g}(\alpha\theta-\alpha^{/}\gamma)-\frac{1}{k_1}\left(\nu-\frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q}, \quad (8)$$

$$\nabla.\vec{q}=0 \quad (9)$$

$$E\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta, \quad (10)$$

$$E^{/}\frac{\partial\gamma}{\partial t}=\beta^{/}w+\kappa^{/}\nabla^2\gamma. \quad (11)$$

NORMAL MODES ANALYSIS

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w,\theta,\gamma]=[W(z),\Theta(z),\Gamma(z)]\ \exp(ik_x x + ik_y y + nt), \quad (12)$$

where k_x, k_y are the wave numbers along the x- and y-directions respectively, k= ($\sqrt{k_x^2+k_y^2}$) is the resultant wave number and n is the growth rate which is, in general, a complex constant. $W(z),\Theta(z)$ and $\Gamma(z)$ are the functions of z only.

Using (12), equations (8)-(11), within the framework of Boussinesq approximations, in the non-dimensional form transform to

$$(D^2-a^2)\left[\frac{F}{P_l}(D^2-a^2)-\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\right]W=Ra^2\Theta-R_s a^2\Gamma \quad (13)$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-W, \quad (14)$$

$$(D^2-a^2-E^{/}p_3\sigma)\Gamma=-W, \quad (15)$$

Where we have introduced new coordinates (x',y',z') = (x/d, y/d, z/d) in new units of length d and $D=d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a=kd,\sigma=\frac{nd^2}{\nu}$, $p_1=\frac{\nu}{\kappa}$, is the thermal Prandtl number;, $p_3 = \frac{\nu}{\kappa'}$ is the thermosolutal Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\mu'/(\rho_0 d^2)}{\nu}$, is the dimensionless couple-stress parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number and $R_s = \frac{g\alpha'\beta' d^4}{\kappa'\nu}$ is the thermosolutal Rayleigh number. Also we have Substituted $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$, $\Gamma = \frac{\beta' d^2}{\kappa'}\Gamma_\oplus$ and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z = 0 and z = 1 respectively, as the case may be, and maintained at constant temperature and solute concentration. Then the perturbations in the temperature and solute concentration are zero at the boundaries. The appropriate boundary conditions with respect to which equations (13)--(15), must possess a solution are W = 0 = $\Theta = \Gamma$, on both the horizontal boundaries, DW = 0, on a rigid boundary, $D^2W = 0$, on a dynamically free boundary, (16)

Equations (13)-(15), along with boundary conditions (16), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W, Θ and Γ satisfy $W(0) = 0 = W(1)$, $\Theta(0) = 0 = \Theta(1)$ and $\Gamma(0) = 0 = \Gamma(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality Schultz [23]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz\,;\quad \int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz$$

And $\int_0^1 |D\Gamma|^2 dz \geq \pi^2 \int_0^1 |\Gamma|^2 dz$ (17)

Further, for $W(0) = 0 = W(1)$, Banerjee et al $[24]$ have shown that

$$\int_0^1 |D^2W|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz. \qquad (18)$$

MATHEMATICAL ANALYSIS

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz \leq \frac{1}{\pi^2(\pi^2 + a^2)} \int_0^1 |DW|^2 dz.$$

Proof: Multiplying equation (14) by Θ^* (the complex conjugate of Θ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Θ namely $\Theta(0) = 0 = \Theta(1)$, it follows that

$$\int_0^1 \left\{|D\Theta|^2 + a^2|\Theta|^2\right\} dz + E\sigma_r p_1 \int_0^1 |\Theta|^2 dz = \text{Real part of}$$

$$\left\{\int_0^1 \Theta^* W dz\right\}, \quad \leq \left|\int_0^1 \Theta^* W dz\right| \quad \leq \int_0^1 |\Theta^* W| dz$$

$$\leq \int_0^1 |\Theta^*||W| dz, \leq \int_0^1 |\Theta||W| dz$$

$$\leq \left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \qquad (19)$$

(Utilizing Cauchy-Schwartz-inequality),

So that by using inequality (17) and the fact that $\sigma_r \geq 0$, we obtain from the above that

$$(\pi^2 + a^2)\int_0^1 |\Theta|^2 dz \leq \left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}},$$

And thus, we get

$$\left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{(\pi^2 + a^2)} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \qquad (20)$$

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Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (19) on utilizing (20) and (17), gives

$$\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz \leq \frac{1}{\pi^2(\pi^2+a^2)}\int_0^1 |DW|^2 dz ,$$

(21)

This completes the proof of lemma 1.

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz \leq \frac{1}{\pi^2(\pi^2+a^2)}\int_0^1 |DW|^2 dz .$$

Proof: Multiplying equation (15) by Γ^* (the complex conjugate of Γ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, it follows that

$$\int_0^1 \left\{|D\Gamma|^2 + a^2|\Gamma|^2\right\}dz + E'\sigma_r p_3 \int_0^1 |\Gamma|^2 dz = \text{Real part of} \left\{\int_0^1 \Gamma^* W dz\right\}, \leq \left|\int_0^1 \Gamma^* W dz\right| \leq \int_0^1 |\Gamma^* W| dz \leq \int_0^1 |\Gamma^*||W| dz ,$$

$$\leq \int_0^1 |\Gamma||W| dz \leq \left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}} ,\quad (22)$$

(Utilizing Cauchy-Schwartz-inequality),

So that by using inequality (17) and the fact that $\sigma_r \geq 0$, we obtain from the above that

$$(\pi^2+a^2)\int_0^1 |\Gamma|^2 dz \leq \left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}} ,$$

And thus, we get

$$\left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{(\pi^2+a^2)} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}} ,$$

(23)

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (22) on utilizing (23) and (17), gives

$$\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz \leq \frac{1}{\pi^2(\pi^2+a^2)}\int_0^1 |DW|^2 dz ,$$

(24)

This completes the proof of lemma 2.

Now we prove the following theorems:

Theorem 1: : If R$\rangle$ 0, $R_s \rangle 0$, F$\rangle$ 0, $P_l \rangle 0$, $p_1 \rangle 0$, $p_3 \rangle 0$, $\sigma_r \geq 0$, $\sigma_i \neq 0$ and $R_s \geq R$, then the necessary condition for the existence of non-trivial solution (W, Θ, Γ) of equations (13) – (15), together with boundary conditions (16) is that

$$R_s \rangle 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\} .$$

Proof: Multiplying equation (13) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2-a^2\right)^2 W dz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 W^*\left(D^2-a^2\right)W dz$$

$$= Ra^2 \int_0^1 W^*\Theta dz - R_s a^2 \int_0^1 W^*\Gamma dz ,\quad (25)$$

Taking complex conjugate on both sides of equation (14), we get

$$\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* = -W^* ,\quad (26)$$

Therefore, using (26), we get

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz ,$$

(27)

Taking complex conjugate on both sides of equation (15), we get

$$\left(D^2 - a^2 - E' p_3 \sigma^*\right)\Gamma^* = -W^* ,$$

(28)

Therefore, using (28), we get

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$$\int_0^1 W^*\Gamma dz = -\int_0^1 \Gamma\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* dz\,,$$

(29)

Substituting (27) and (29), in the right hand side of equation (25), we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 W^*(D^2 - a^2)W dz =$$

$$-Ra^2\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz$$

$$+ R_s a^2\int_0^1 \Gamma(D^2 - a^2 - E' p_3\sigma^*)\Gamma^* dz \quad (30)$$

Integrating the terms on both sides of equation (30) for an appropriate number of times and making use of the appropriate boundary conditions (16), we get

$$\frac{F}{P_l}\int_0^1 \left\{|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz$$

$$= Ra^2\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2 + Ep_1\sigma^*|\Theta|^2\right)dz - R_s a^2\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2 + E' p_3\sigma^*|\Gamma|^2\right)dz \,, \quad (31)$$

now equating real and imaginary parts on both sides of equation (31), and cancelling $\sigma_i (\neq 0)$ throughout from imaginary part, we get

$$\frac{F}{P_l}\int_0^1 \left\{|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz$$

$$= Ra^2\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2 + Ep_1\sigma_r|\Theta|^2\right)dz - R_s a^2\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2 + E' p_3\sigma_r|\Gamma|^2\right)dz \,, \quad (32)$$

and

$$\frac{1}{\varepsilon}\int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz = -Ra^2 Ep_1\int_0^1 |\Theta|^2 dz + R_s a^2 E' p_3\int_0^1 |\Gamma|^2 dz$$

(33)

of which the equation (32) can be rearranged in the form

$$\frac{F}{P_l}\int_0^1 \left\{|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz$$

$$= Ra^2\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz - R_s a^2\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz + \sigma_r\left[Ra^2 Ep_1\int_0^1 |\Theta|^2 dz - R_s a^2 E' p_3\int_0^1 |\Gamma|^2 dz\right] \,, (34)$$

The equation (33) together with $\sigma_r \geq 0$, yields the inequality

$$\sigma_r\left[Ra^2 Ep_1\int_0^1 |\Theta|^2 dz - R_s a^2 E' p_3\int_0^1 |\Gamma|^2 dz\right] \leq 0\,, \quad (35)$$

Now, utilizing the inequality (17), we have

$$\int_0^1 \left\{|D\Gamma|^2 + a^2|\Gamma|^2\right\}dz \geq (\pi^2 + a^2)\int_0^1 |\Gamma|^2 dz\,, \qquad (36)$$

While from the equation (33) on utilizing (17), we get

$$\int_0^1 |\Gamma|^2 dz \geq \frac{1}{R_s a^2 E' p_3\varepsilon}\int_0^1 |DW|^2 dz\,, \qquad (37)$$

So that using inequality (37), we can write the inequality (36) as

$$\int_0^1 \left\{|D\Gamma|^2 + a^2|\Gamma|^2\right\}dz \geq \frac{(\pi^2 + a^2)}{R_s a^2 E' p_3\varepsilon}\int_0^1 |DW|^2 dz\,, \quad (38)$$

Now we prove the following theorems:

Now, if permissible let $R_s \geq R$, Then in that case we derive from equation (34) and utilizing the inequalities (17), (18), (21) and (38), we get

$$\left[(\pi^2 + a^2)\left\{\frac{F}{P_l} + \frac{1}{E' p_3\varepsilon}\right\} - \frac{R_s a^2}{\pi^2(\pi^2 + a^2)}\right]\int_0^1 |DW|^2 dz + I_1 \langle 0\,, \qquad (39)$$

Where

$I_1 = \left(\frac{a^2 F}{P_l} + \frac{\sigma_r}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz$, is positive definite. Therefore, we must have

$$R_s \rangle \frac{\pi^2(\pi^2 + a^2)^2}{a^2}\left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\}. \qquad (40)$$

and thus we necessarily have

$$R_s \rangle 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\} \qquad (41)$$

Since the minimum value of $\frac{\pi^2\left(\pi^2 + a^2\right)^2}{a^2}$ is $4\pi^4$ at $a^2 = \pi^2 \rangle 0$.

Hence, if

$\sigma_r \geq 0$ and $\sigma_i \neq 0$, then

$$R_s \rangle 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\}, \qquad (42)$$

And this completes the proof of the theorem.

Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 1, can be put in the form as follow:-

Corollary 1: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal couple-stress fluid configuration of Veronis type in a porous medium heated from below is that, $R_s \leq 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\}$, where R_s is the Thermosolutal Rayliegh number, p_3 is the thermosolutal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the couple-stress parameter, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal couple-stress fluid configuration of Veronis type in a porous medium heated from below, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F, satisfy the inequality $R_s \leq 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\}$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 2: If $(W, \Theta, \Gamma\sigma)$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (13) – (15) together with boundary conditions (16) and

$$R_s \leq 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\},$$

Then $\sigma_i = 0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ if $R_s \leq 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\}$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Veronis type, we can state the above theorem as follow:-

Corollary 3: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal couple-stress fluid configuration of Veronis type in a porous medium heated from below is that the Thermosolutal Rayliegh number R_s, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F must satisfy the inequality $R_s \rangle 4\pi^4 \left\{\frac{F}{P_l} + \frac{1}{E' p_3 \varepsilon}\right\}$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

A similar theorem can be proved for thermosolutal convection in couple-stress Viscoelastic fluid configuration of Stern type in a porous medium as follow:

Theorem 2: If R $\langle 0$, $R_s \langle 0$, F $\rangle$ 0, $P_l \rangle 0$, $p_1 \rangle 0$, $p_3 \rangle 0$, $\sigma_r \geq 0$, $\sigma_i \neq 0$ and $|R| \geq |R_s|$ then the necessary condition for the existence of non-trivial solution (W, Θ, Γ) of equations (13) – (15), together with boundary conditions (16) is that

$$|R| \rangle 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{Ep_1\varepsilon} \right\}.$$

Proof: Replacing R and R_s by $-|R|$ and $-|R_s|$, respectively in equations (13) – (15) and proceeding exactly as in Theorem 1 and utilizing the inequality (24), we get the desired result.

Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 2, can be put in the form as follow:-

Corollary 4: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal couple-stress fluid configuration of Stern type in a porous medium is that, $|R| \leq 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{Ep_1\varepsilon} \right\}$, where R is the Thermal Rayliegh number, p_1 is the thermal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the couple-stress parameter, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal couple-stress fluid configuration of Stern type in a porous medium, cannot manifest itself as oscillatory motions of growing amplitude if the Thermal Rayliegh number R, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the couple-stress parameter F, satisfy the inequality $|R| \leq 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{Ep_1\varepsilon} \right\}$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 5: If $(W, \Theta, \Gamma\sigma)$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (13) – (15) and

$$|R| \leq 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{Ep_1\varepsilon} \right\},$$

Then $\sigma_i = 0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ if $|R| \leq 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{Ep_1\varepsilon} \right\}$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Stern's type, we can state the above theorem as follow:-

Corollary 6: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal couple-stress fluid configuration of Stern type in a porous medium is that the Thermal Rayliegh number R, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the couple-stress parameter F must satisfy the inequality $|R| \rangle 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{Ep_1\varepsilon} \right\}$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid.

CONCLUSIONS

Theorem 1 mathematically established that the onset of instability in a thermosolutal couple-stress fluid configuration of Veronis type, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F satisfy the inequality $R_s \leq 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{E^/ p_3 \varepsilon} \right\}$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

The essential content of the theorem 1, from the point of view of linear stability theory is that for the thermosolutal configuration of Veronis type of couple-stress fluid of infinite horizontal extension, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $R_s \leq 4\pi^4 \left\{ \frac{F}{P_l} + \frac{1}{E^/ p_3 \varepsilon} \right\}$, and in particular PES is valid.

The similar conclusions are drawn for the thermosolutal configuration of Stern type of couple-stress fluid of infinite horizontal extension, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid from Theorem 2.

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