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A CASE STUDY ON ANT COLONY OPTIMIZATION TECHNIQUE

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ABSTRACT

Ant colony optimization (ACO) is one of the most recent approaches for approximate optimization. The inspiring source of ant colony is the foraging behaviour of real ant colonies. This behaviour is search for inexact solutions to discrete optimization problems, to continuous optimization problems and also consider to prime problems in scheduling, such as make span and time balancing. At the core of this action is the indirect communication between the ants by means of chemical pheromone trails, which entitle them to find short paths between their nests to food sources. This attribute of real ant colonies is exploited in Ant colony optimization algorithms in order to resolve discrete optimization problems.

The objective of this paper deals with minimizing the travelling salesman path in a unknown city environment with the practicability of outsourcing certain jobs. The problem addressed in this paper by means of the evolution of an ant colony optimization based algorithm. This new algorithm, here named as travelling on shortest path by ant colony algorithm. The results show that this new approach can be used to solve the problem systematically and in a short computational time.

Key words- Ant colony optimization, Travelling salesman problem, Pheromone

Introduction

Optimization is the mathematical discipline which is concerned with finding the maxima and minima of functions, possibly subject to constraints. There are many optimization techniques based on number of objectives to be optimized such as single objective optimization techniques, multi objective optimization techniques and non-traditional optimization techniques. Single objective optimization techniques are used to solve the problems having single objective function with or without constraints and multi objective optimization techniques are used to solve the problems having more than one objective functions with or without constraints.

Multi-objective optimization brings the optimization literature close to practice, as in a real world design or operation; often there exists not one, but a few a goal or objectives that must be considered simultaneously. Generally the objectives are in conflict in nature, meaning that the optimal solution of one objective is not the optimal solution of any other objective.

1.1 Ant colony optimization (ACO)

ANT colony optimization [1] technique developed by Marco Dorigo in 1991 is based upon the real ant behaviour in finding the shortest path between the nests to food. They attained this by indirect communication by a substance called pheromone which shows the trail of the ant. Ant utilizes heuristic information of its own knowledge the smell of the food and the decision of the path travelled by the other ants using the pheromone content on the path. The role of the pheromone is to guide other ants towards the food.

Ant has the capability of searching the food from their nest with the shortest path without having any visual clues. At a given point where there are more than one path to extend to their food then ants distribute themselves on disparate paths also the path and lay pheromone trace on that path to return on same path. Thus the path with minimum distance will acquire more pheromone as compared to other paths as the ants will return faster from that path comparative to the other path. So the new ants coming in the find of food will move with probability towards the path having higher pheromone content as compared to the path having lower pheromone content and in the end all the ants will directed towards the same path with the minimum shortest path to their food. Now figure 1 shows the behaviour of ants moving from the upward direction will return early as compared to the ants moving from the downward direction so the pheromone content in the upward orientation is more as compared to the downward direction due to that in the end all the ants will start going towards the upward direction which is the shortest path to their food [2].

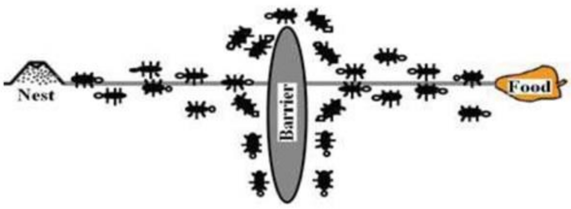

Fig. 1 Behaviour of real Ants

ACO can be used to solve combinatorial optimization problems such as Travelling Salesman Problem (TSP), Vehicle Routing, Quadratic Assignment, Graph Colouring, Project Scheduling, Multiple Knapsack etc. maximum of the problems are NP-hard problem i.e. they take exponential time complexity in their worst case [1].

1.2 Ant System Algorithm

Ant system was the first algorithm presented under ACO to solve TSP (Travelling Salesman) problem. In this algorithm all the ants (traveller) update their pheromone values after completing a solution. In the construction of a solution an ant chooses next node to be visited utilizing a stochastic mechanism [2].

An ant k at city i has not visited set of cities S_p then P_{ij} be the probability to visit edge k after edge i.

$$P_{i\,j}^{k} = \frac{\tau_{ij}^{\alpha}\ \eta_{ij}^{\beta}}{\sum_{j \in Sp} \tau_{ij}^{\alpha} \eta_{ij}^{\beta}} \quad \text{if } j \in S_p \qquad \text{.... (1)}$$

S_p represents the set of cities which has not been visited yet and to be visited again so that the probability of the ant visiting a city which has already visited becomes 0.

Where τ_{ij} is the pheromone content on the edge joining node i to j.

η_{ij} represents the heuristic value which is inverse of the distance between the city i to j, which is given by,

$$\eta_{ij} = \frac{1}{d_{ij}} \qquad \text{........ (2)}$$

Where d_{ij} is the distance between the cities i to j.

α and β represent the dependency of probability on the pheromone content or the heuristic value respectively. Increasing the value of α and β may vary the convergence of ACO [2].

After solution construction we have to update the pheromone accordingly, as follows:

$$\tau_{ij} \leftarrow (1\text{-}\zeta)\ \tau_{ij}^{k} + \sum_{k=1}^{m} \Delta\, \tau_{ij}^{k} \qquad \text{......... (3)}$$

Where ρ is the evaporation rate, *m* is the number of ants and Δ is the quantity of pheromone laid on edge (i,j) by an ant k [2].

$\Delta\tau_{ij}^{k} = \frac{Q}{L_k}$, if ant k uses edge (i,j) in its tour, 0, otherwise. .... (4)

Where Q is a constant and L_k is the length of the tour constructed by an ant k. literature review

Title	Author & Publication Details	Methodology Used	Key Findings
Parallel implementation of travelling salesman	Gaurav Bhardwaj, Manish Pandey &	Ant colony optimization	Basic introduction of Control parameters (α,β,ζ)with

problem	International journal of Computer Applications Technology and Research volume 3, 385-389, 2014	method	optimal value is obtained
Integrated scheduling of production and distribution to minimize total cost using improved ant colony optimization method	Ba Yi Cheng, Joseph Y, Kai Li, T. Leung" & computers and industrial engineering, 2015	Improved ant colony optimization method	They developed two model production and distribution part to minimize cost of both. The performance of heuristic is excellent while running time is no more than 5sec for 200 jobs for distribution part
Application ACO on generalized Travelling Salesman Problem	Kan Jun man, Zhang Yi & Energy Procrdia, 319-325,2012	Ant colony optimization method	The quality solution of speed & convergence has been enhanced
Ant Colony Optimization : Introduction and recent trends	Christian Blum & Physics of life reviews,353-373, 2005	Ant Colony Optimization	ACO can be applied to discrete optimization problem also deal with biological inspiration of ant colony and reduction of search space is enhanced

Case Study

Travelling Salesman Problem solving by using Ant Colony optimization

Travelling Salesman [2] Problem represents a set of problem called combinatorial optimization problem. In TSP a salesman is given a map of cities and he has to visit all the cities exactly once and return back to the starting city with the minimum cost length tour of all the possible tour present in that map. Hence the total number of possible tour in a graph with n vertices is (n-1)!

There are various approaches to solve TSP. Classical approach to solve TSP are dynamic programming, branch and bound which uses heuristic and exact method and results into exact solution. But as we know TSP is an NP-hard problem so the time complexity of these algorithms are of exponential time. So they can solve the small problem in optimal time but as compared to the large problem time taken by these algorithms are quite high. So no classical approach can solve this type of problem in reasonable time as the size of the problem increases complexity increases exponentially. To overcome this different based on natural and population Ant Colony Optimization technique is introduced [2].

1) Hamiltonian Path (Traceable Path)

A Hamiltonian path in an undirected graph which visits each vertex exactly once. A Hamiltonian cycle in undirected graph which visits each vertex exactly once and also returns to the starting vertex. Determining whether such paths and cycles exist in graphs is the Hamiltonian path problem which is NP completes [5].

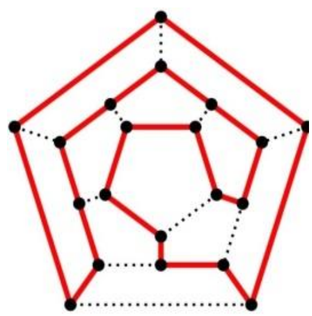

Fig. 2 A graph representing the vertices edges and faces of a dodecahedron with a Hamilton cycle shown by Red colour edges.

2) Problem

2.1) Calculate shortest path between Cities given in the below table.

TABLE I Distance between cities

Distance between cities	Units
AB	100
AD	50
AE	70
AC	60
BE	60
BC	60
CD	90
CE	40
DE	150

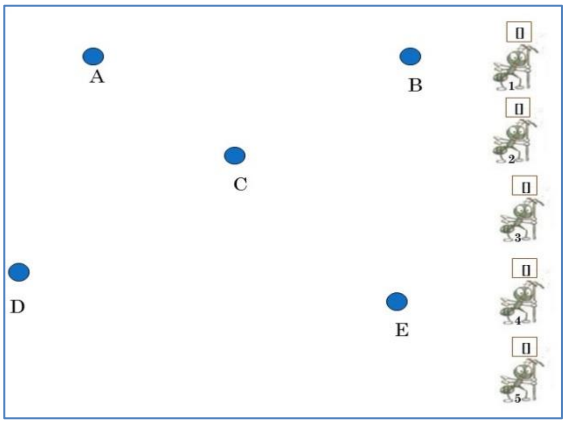

Fig. 3 Here m=n; m= number of ants (traveller)
n= number of cities

Method for solving Travelling Salesman Problem by Ant colony optimization

1) Iteration 1

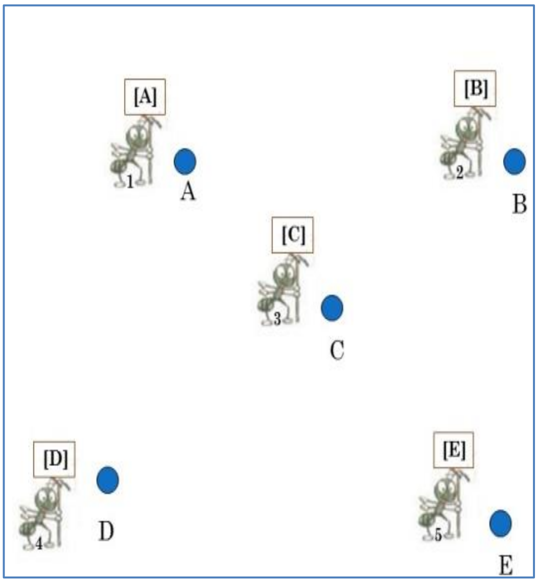

Fig. 4 Each ant is assigned to different city

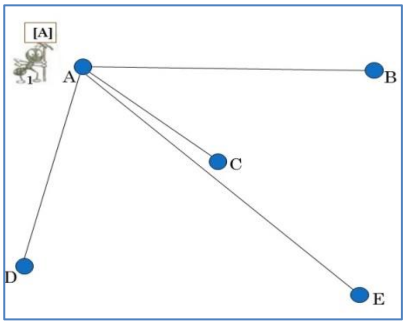

Fig. 5 A1 has four different options

Ant (traveller) A1 has four different option i.e. AB=100, AC=60, AD=50 and AE=70 unit,

A1 will choose path by a probabilistic value according to equation (1). For particular $P^{A}_{A\,D}$ given by,

$$P^{A}_{A\,D} = \frac{0.08^{0.2}_{AD}\ (\frac{1}{50})^{0.6}_{AD}}{\sum_{D\epsilon Sp} 0.08^{0.2}_{AD}(1/50)^{0.6}_{AD}} = 0.25$$

Value of α=0.2, β=0.6

Similarly we can find P^{k}_{AB}(t) = 0.38, P^{k}_{AC}(t) = 0.47 and P^{k}_{AE}(t) = 0.31

The amount of pheromone on edge (A, B), edge (A, C), edge (A, D) and edge (A, E) are $\tau^{k}_{A,B}$(t), $\tau^{k}_{A,c}$(t), $\tau^{k}_{A,D}$(t) and $\tau^{k}_{A,E}$(t) respectively with same initial value 0.2

The value of η(A, B), η(A, C), η(A, D) and η(A, E) are 1/100, 1/60, 1/50, 1/70 respectively, according to equation (2).

Depending upon random number and smallest value A1 choose AD as path. P^{A}_{AD}=0.25.

According to equation (3) Pheromone decay,

τ_{AD}(t) = (1-ζ) τ^{k}_{ij}(t) + $\sum^{m}_{k=1} \Delta \tau^{k}_{ij}$(t)

(Assume that ζ = 0.6)

TABLE II Evaporation rate in Iteration 1

Iteration	Path	Decay Rate
1	AD	0.08

2) **Iteration 2**

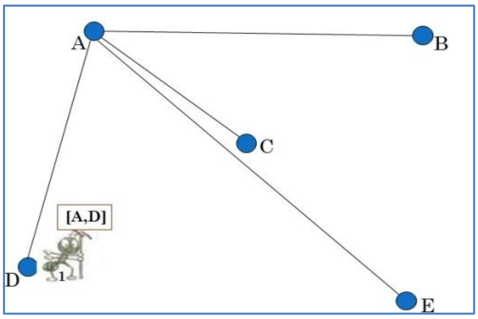

Fig.6 Path travelled by ant is [A, D]

Ant (traveller) A1 has three different option i.e. DB=70, DC=90 and DE=150 unit,

Again A1 will choose path by a probabilistic value, according to equation (1).

The amount of pheromone on edge (D, B), edge (D, C) and edge (D, E) are $\tau^k_{D,B}$(t), $\tau^k_{D,c}$(t) and $\tau^k_{D,E}$(t) respectively with same initial value 0.2

The pheromone on path AD after first iteration will be updated by,

$\Delta\tau^k_{ij}$ = $\frac{Q}{L_k}$ (t) if (i, j) €T^k(t) else 0, according to equation (4).

$\Delta\tau^A_{AD}$= 100/50

where, Q=100, L^k=50

$\Delta\tau^A_{AD}$= 2.

Value of α=0.2, β=0.6

For particular $P^A_{D\ C}$ given by,

$$P^A_{D\ C} = \frac{0.0032^{0.2}_{DC}\ (\frac{1}{90})^{0.6}_{DC}}{\sum_{D\epsilon Sp} 0.0032^{0.2}_{DC}(1/90)^{0.6}_{DC}} = 0.32$$

Now similarly we can find probabilities P^k_{DB}(t) = 0.38, P^k_{DC}(t) = 0.32, and P^k_{DE}(t) = 0.43 respectively.

Depending upon random number and smallest value A1 choose DC as path. P^A_{DC}=0.32

According to equation (3) Pheromone decay,

τ_{DC}(t) = (1-ζ) τ^k_{AD}(t) + $\sum^m_{k=1} \Delta\, \tau^k_{AD}$(t)

(Assume that ζ = 0.6)

TABLE III Evaporation rate in Iteration 2

Iteration	Path	Decay Rate
2	DC	0.032

3) **Iteration 3**

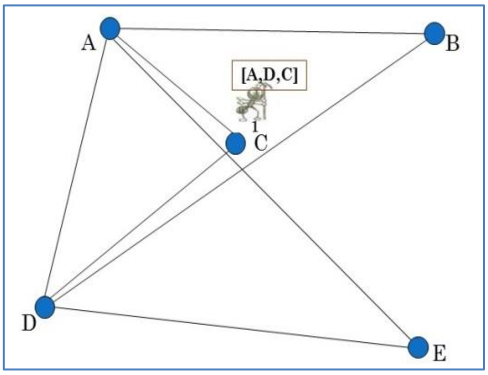

Fig.7 Path travelled by ant is [A, D, C]

Now, Ant (traveller) A1 has another two different option i.e. CE=40, CB=60 unit,

Again A1 will choose path by a probabilistic value, according to equation (1).

The amount of pheromone on edge (C, B) and edge (C, E) are $\tau^k_{C,B}$(t), and $\tau^k_{C,E}$(t) respectively with same initial value 0.2.

The pheromone on path DC after second iteration will be updated by,

$\Delta\tau^k_{ij}$ = $\frac{Q}{L_k}$ (t) if (i, j) €T^k(t) else 0, according to equation (4).

$\Delta\tau^A_{DC}$= 100/140

where, Q=100, L^k=50+90=140

$\Delta\tau^A_{DC}$= 0.7142

Value of α=0.2, β=0.6

For particular $P^A_{C\ E}$ given by,

$$P^A_{C\ E} = \frac{0.0128^{0.2}_{CE}\ (\frac{1}{40})^{0.6}_{CE}}{\sum_{D\epsilon Sp} 0.0128^{0.2}_{CE}(1/40)^{0.6}_{CE}} = 0.5$$

Now similarly we can find probabilities P^k_{CB}(t) = 0.61 and P^k_{CE}(t) = 0.5 respectively.

Depending upon random number A1 choose CE as path. P^A_{CE}=0.5

According to equation (3) Pheromone decay,

τ_{CE}(t) = (1-ζ) τ^k_{DC}(t) + $\sum^m_{k=1} \Delta\, \tau^k_{DC}$(t)

(Assume that ζ = 0.6)

TABLE IV Evaporation rate in Iteration 3

Iteration	Path	Decay Rate
3	CE	0.0128

4) **Iteration 4**

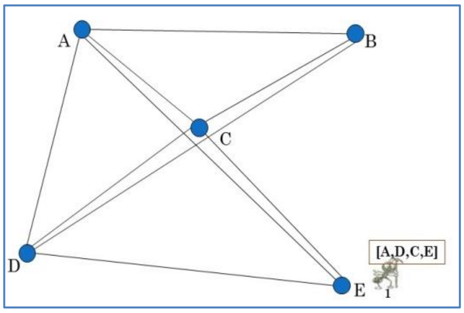

Fig.8 Path travelled by ant is [A, D, C, E]

Now, Ant (traveller) A1 has only one path i.e. EB=60 unit, A1 has already visited city A,C and D.

Again A1 will choose path by a probabilistic value, according to equation (1).

The amount of pheromone on edge (E, B) is $\tau^k_{E,B}$(t) with initial value 0.2.

The pheromone on path CE after third iteration will be updated by,

$\Delta\tau^k_{ij}$ = $\frac{Q}{L_k}$ (t) if (i, j) €T^k(t) else 0, according to equation (4).

$\Delta\tau^A_{CE}$= 100 where, Q=100, L^k=50+90+40=180

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$\Delta\tau_{CE}^{A}$= 0.5555

Value of α=0.2, β=0.6

For particular $P_{E\,B}^{A}$ given by,

$$P_{E\,B}^{A} = \frac{0.00512_{EB}^{0.2}\ (\frac{1}{60})_{EB}^{0.6}}{\sum_{D \in Sp} 0.00512_{EB}^{0.2}(1/60)_{EB}^{0.6}} = 0.5$$

Depending random number and smallest value A1 choose EB as path. P_{EB}^{A}=0.5

According to equation (3) Pheromone decay,

τ_{EB}(t) = (1-ζ) τ_{CE}^{k}(t) + $\sum_{k=1}^{m} \Delta\,\tau_{CE}^{k}$(t)

(Assume that ζ = 0.6)

TABLE V Evaporation rate in Iteration 4

Iteration	Path	Decay Rate
4	EB	0.00512

5) Iteration 5

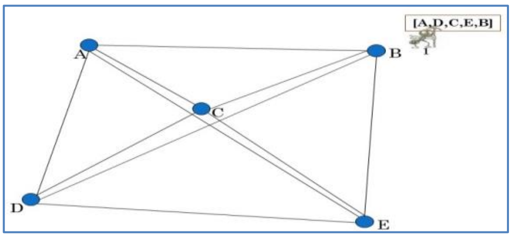


Fig.9 Path travelled by ant is [A, D, C, E, B]

Now, Ant (traveller) A1 has only one path i.e. BA=100 unit, A1 has already visited all other city and return to home city.

Again A1 will choose path by a probabilistic value, according to equation (1).

The amount of pheromone on edge (B, A) is $\tau_{B,A}^{k}$(t) with initial value 0.2

The pheromone on path EB after fourth iteration will be updated by,

$\Delta\tau_{ij}^{k}$ = $\frac{Q}{L_k}$ (t) if (i, j) €T^k(t) else 0, according to equation (4).

$\Delta\tau_{CE}^{A}$= 100/240; where, Q=100, L^k=50+90+40+60=240

$\Delta\tau_{CE}^{A}$= 0.4066

Value of α=0.2, β=0.6

And probability P_{BA}^{A}(t) is given by,

$$P_{B\,A}^{A} = \frac{0.002048_{BA}^{0.2}\ (\frac{1}{100})_{BA}^{0.6}}{\sum_{D \in Sp} 0.002048_{BA}^{0.2}(1/100)_{BA}^{0.6}} = 0.8$$

Depending upon random number and smallest value A1 choose BA as path. P_{BA}^{A}=0.8

According to equation (3) Pheromone decay,

τ_{BA}(t) = (1-ζ) τ_{CE}^{k}(t) + $\sum_{k=1}^{m} \Delta\,\tau_{CE}^{k}$(t)

(Assume that ζ = 0.6)

TABLE VI Evaporation rate in Iteration 5

Iteration	Path	Decay Rate
5	BA	0.002048

6) Iteration 6

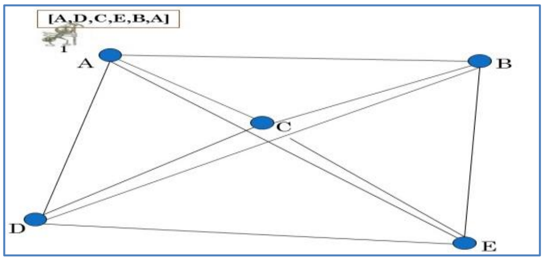


Fig.10 Path travelled by ant is [A, D, C, E, B, A]

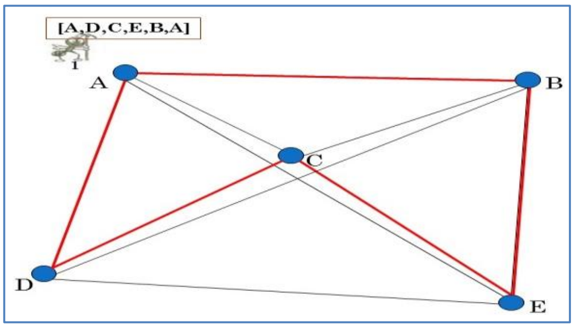


Fig. 11 optimal path for ant A1

7) Path for each ant and path length

L1[ADCEBA]=50+90+40+60+100

=340

Similarly path for other Ants are presented [2] here with optimal path length,

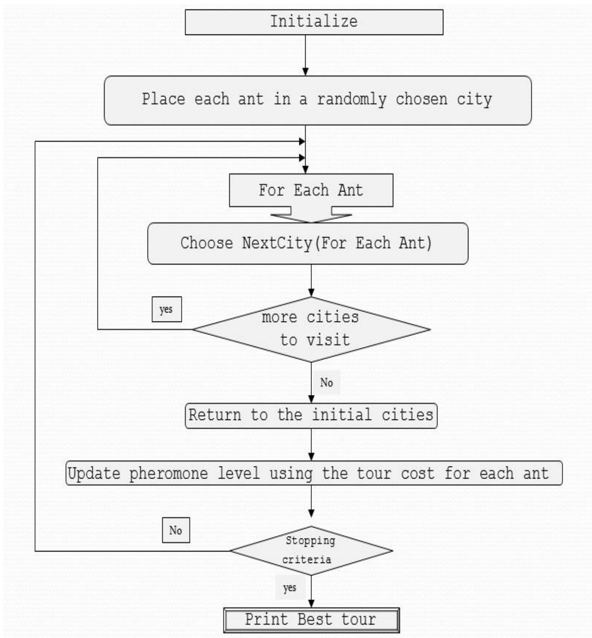
L2[BCDAEB]=60+90+50+70+60=330

L3[CBEDAC]=60+60+150+50+60=380

L4[DEABCD]=150+70+100+60+90=470

L5[EABCDE]=70+100+60+90+150=470

Flow chart for travelling salesman problem



Conclusion

Based on the Ant colony optimization technique, following conclusion can be drawn,

- ACO is a recently proposed metaheuristic approach for solving hard combinatorial optimization problems (NP HARD Problems).
- The cumulated search experience is taken into account by the adaptation of the pheromone trail.
- ACO Shows great performance with the "ill-structured" problems like network routing.
- In ACO Local search is extremely important to obtain good results.

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