



A Brief Review of Dilemma Zone and Methods for Protection at Signalized Intersection

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ABSTRACT

This paper not only demonstrates the development of the concept of a dilemma zone but also introduced its mitigation. Dilemma zone (DZ) usually happens at high-speed and signalized intersection, where a vehicle neither cannot stop safely before the stop line nor clear the intersection before the onset of the red light. In that case, the driver is involved in a potential hazard. Because of high speed, any decision acting on driver behavior may results in undesirable hazard. Stopping suddenly may making vehicle suffering rear-end collision and accelerating to pass through the intersection could incur rectangular crash. Thus, determining how to prevent the vehicle from being trapped in a DZ has raised wide attention. This paper briefly reviews previous study of DZ, Dilemma hazard model and DZ's protection methods. At last, future work concerning DZs are discussed.

Keywords—dilemma zone; signalized intersection; driver behaviour; DZ protection

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I. Introduction

As we all know yellow light plays an important role in traffic conflict, the “yellow interval” is usually the key point to cause a conflict at a signalized intersection. Traffic signal control once again draws public's attention since Gazis, Herman and Maradudin (GHM) gave the concept of yellow light dilemma zone[1]. An area where drivers can't stop safely or pass through the intersection before the start of red light is called dilemma zone. Sheffi and Mahmassani further expounded that “The dilemma refers to the drivers' decision to proceed through the intersection or to stop when the signal indication changes from green to amber” [2]. Since then, DZ and yellow signal have been widely studied by researchers.

Given that vehicles facing with DZ have great probability to get in trouble with rear-end collision or rectangular crash, the number of

vehicles in dilemma zone gradually becomes the surrogate measurement of traffic safety [3]. Dilemma zone hazard model has been proposed, which determines the risk of traffic conflict for an individual vehicle in dilemma zone [3][4]. Many researchers are devoted into reducing the likelihood of vehicles' being trapped in dilemma zone. According to formula given by Gazis, which took use of the speed limit, maximum acceleration/ deceleration rates, the dilemma zone can be eliminated by setting the yellow interval. However, dilemma zone does exist and researchers like Liu explained why dilemma zone exists in reality [5]. The approaches to remove DZ can be divided into categories: resetting the yellow light signal [1], advanced warning system [6], advance detector system[3][7-9], and in-vehicle warning system [10-12].we will describes newest methods for Dilemma Zone protection. This paper contains five sections

totally. Section one gives the introduction of this topic; Section 2 shows the concepts of two types of DZ, relevant DZ models and key factors; section 3 explains dilemma hazard model and its background; section 4 places the primary emphasis on DZ protection, where existing methods for protection are discussed; At last section, draw a conclusion and give future work direction.

II. The yellow light dilemma zone

Type I Dilemma Zone

Dilemma zones are usually divided into two types, Type I is aforementioned and defined by GHM in 1960, showed in Fig.1. Dilemma zone refers to an area where vehicles can neither pass through the intersection nor stop safely with approaching speed. To the contrary, the option zone refers to an area where the vehicle can be safe no matter whether it chooses to stop or not. Actually, the type I and type II classification are definitely proposed by Koonce and Urbanik.

According to GHM model proposed by Gazis, Denos, we can calculate the dilemma zone with constant speed approach and typical value of acceleration and deceleration rates[1]. The formulas are as follows.

$$X_s = v_0 \delta_1 + \frac{v_0^2}{2a_1} \tag{1}$$

$$X_c = v_0 \tau + \frac{a_1}{2} (\tau - \delta_2)^2 - W - L \tag{2}$$

where v_0 is vehicle's approaching speed; δ_1 is driver's perception reaction time(PRT) for stopping and δ_2 is driver's PRT for clearing; a_1 and a_2 are the vehicle's rate acceleration and deceleration rate, respectively; τ is yellow light interval time; W is the width of the intersection and L is the length of the vehicle.

As shown in Fig.1, there may exist one of the two kinds of special zones: the option zone and dilemma zone. The former appears that X_s is smaller than X_c when where a vehicle can safely stop or clear the intersection. However, because of the limit of speed and distance as well as duration time of yellow interval, the second zone turns up undesirably, where X_s is greater than X_c and a vehicle can't stop promptly or clear the intersection timely. Type I DZ exists with a length $(X_s - X_c)$.

According to GHM model, assuming that a crossing vehicle does not accelerate, the DZ can be eliminated by adjusting the yellow light interval to set $(X_s - X_c)$ to zero as expressed in (3).

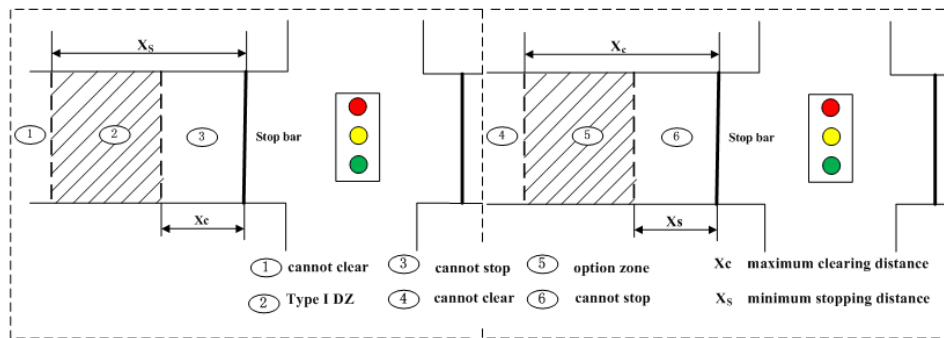


Fig.1 Type I dilemma zone and option zone.

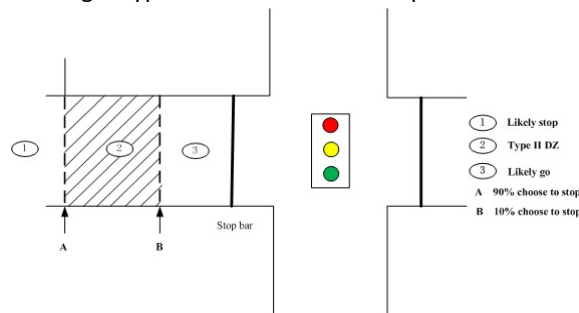


Fig.2 Type II dilemma zone and option zone

$$\tau = \delta + \frac{v_0}{2a_1} + \frac{L+W}{v_0} \quad (3)$$

Some researchers divided the τ into yellow interval $y = \delta + v_0/2a_1$ and the all-red clearance interval $r = (L+W)/v_0$ on condition that driver's PRT is same for stopping or crossing. However, Chiu Liu didn't think there is a need to partition a yellow interval into a "yellow" and a "red" clearance [13].

TABLE I LENGTH OF TYPE I DILEMMA ZONE

Acceleration Rate(m.s ⁻¹)	Lever of Comfort
<0.315	Comfortable
0.315~0.63	Slightly uncomfortable
0.63~1.0	A little uncomfortable
1.25~2.5	Uncomfortable
>2.0	Very imcomfortable

The GHM model and its extensions are essentially kinematic models and the drivers' psychology is ignored. After the type I DZ has been given, several other notable extended models were reported. For example, Liu and Herman made assumption that acceleration of a vehicle decreases linearly with its speed throughout the acceleration maneuver[14]. However, some vehicles may accelerate or decelerate heavily to escape DZ, which doesn't meet the assumption of GHM model that vehicles will firstly choose to stop, if they can, at the onset of yellow light[14].

A. Key Factors in Type I DZ

According to GHM model, it's easy to see the approaching speed and acceleration rate as well as deceleration rate are the most controversial factors in type I DZ's calculation. In most common engineering practices, these parameters are typically assumed having constant values. In addition, the acceleration rate is usually equal to 0 ft/s²(a₂=0) and the approaching speed is always identical to the speed limit. Maximum acceleration/deceleration rate needs to be constrained and acceleration/deceleration rate have to abide by physical law. Gazis suggested to use 0.5~0.8g as the maximum deceleration rate[1]; in the ITE's (Institute of Transportation Engineers)handbook, it equals to -10ft/s²[15], Li took -10ft/s²,too[3]. Liu and Herman proposed a linearly decreased model between accelerate and

instantaneous speed[13].About deceleration rate, accordingly, Gazis recommended 0.3~0.5g as the maximum acceleration rate; ITE and LI uses 15 ft/s². Li discussed the random acceleration / deceleration rates were functions of TTI(time to intersection)[3].Some literature gives the comfortable acceleration/deceleration rates, shown in Tab.1,where a_{comfor+}= 0.315m/s², =-3.048m/s².

Drivers' risk preference should also be considered in type I DZ. Li and Abbas put forward that perception-reaction was an indicator of drivers' aggressiveness and that the more aggressive a driver was, the shorter a PRT he/she would have[3].Reported PRT(δ) include: 1.0s[15]; 1.13s[16]; 0.9s[17]; 1.86s-2.32s[5].In most study, 1.0s is often used as PRT. Zhang calculated the lengths of dilemma zone with different duration of yellow light and a₁, a₂ are equals to 0ft/s², 10ft/s² and 1.0s[18].

B. Type II Dilemma zone

Type II DZ first appeared in a committee report given by Parsonson, which is based on probability[19].As he put it, the dilemma zone was actually an option zone where the drivers choose to stop or cross the intersection. The concept of Type II DZ is widely accepted is given by Zegeer, who defined the dilemma zone as "the road segment where more than 10% and less than 90% of the drivers would choose to stop"[8].As Zegeer and Deen defined, the beginning of the zone is the distance from the stop line at which 90% of all drivers would stop if all drivers would stop if presented a yellow indication.

Type II dilemma zone is an area based on drivers' decision thus is also called decision zone or indecision zone, seen in Fig.2. Based on possibility, the Type II dilemma is more widely used in study with measured data.Fig.3 shows some references about the two types. We can see that Type II dilemma zone appears much more frequently in DZ protection system[3].

C. Factors and Models for Type II DZ

Type II DZ usually uses the relationship between TTI and probability to stop to build model. The range of TTI changes in different studies. Change gave the TTI ranging from 3s to 6s[20];according to Bonneson, TTI's range is

2.5s~5.5s[7].Age may influence the value of TTI. Rakka designed an experiment and 60 participants were included. As a result, for drivers under 40, TTI's range is 1.85s~3.9s, while 1.5s~3.2s for drivers older than 70[21]. That's to say age of drivers will affect the decision at the onset of yellow light. There are other factors influence type II DZ, too. Vehicle's approaching speed and distance to stop line would have a difference to drivers' decision[22]; Elmitiny found that vehicles' type would affect probability of stopping [23]. Therefore, it can be seen that type II DZ is dynamic with different vehicles and drivers[24]. Stop-or-cross decision is random while follows a certain probability distribution. The most common models for type II DZ are probit models and logistic models. Probit model may contain binary probit model[2-3], ordered probit model[5]. Logistic models maybe consist of binary logistic model[18], fuzzy sets-binary logistic model and fuzzy logic model[25-26].

Binary probit model is one of the most useful model in analysing dilemma zone problem. According to Sheffi, the probability that a random driver would choose to stop, $P_{STOP}(T)$, is given by the probit equation[2]:

$$T = t + \xi \tag{4}$$

$$T_{cr} = t_{cr} + \zeta \tag{5}$$

$$P_{STOP}(T) = pr\{T_{cr} < T\} = \Theta\left(\frac{t - t_{cr}}{\sigma}\right) \tag{6}$$

In (4), where t is the measured time to stop line at the constant speed; ξ is a random variable distribution across the drivers' population and $\epsilon \sim N(0, \delta_\epsilon^2)$. In (5), T_{cr} is a critical value; if T is less than T_{cr} , then the driver would choose to stop, where $\zeta \sim N(0, \delta_\zeta^2)$ across the driver population. In (6), $\Theta(\cdot)$ denotes the standard cumulative normal function and $\sigma = \sqrt{(\delta_\epsilon^2 + \delta_\zeta^2)}$. Then, a likelihood function was used to estimate the model parameters and σ . The boundaries of type II DZ can be computed after speed was taken into model.

Another model often be used is binary logistic regression model. Most researchers use the binary logistic regression because driver only has two options in type II DZ. The form of this model is given by Gates in 2007[17].

$$P_{STOP}(T) = \frac{1}{1 + e^{-U_i}} \tag{7}$$

$$U_i = a + bX_i + cX_iX_j \tag{8}$$

where P_{STOP} is the probability of i th driver choosing to stop; U_i donates utility; a is constant value; X_i, X_j are the explanatory variable and sometimes contain artificial variable.

For example, to study whether green signal countdown display would affect the type II DZ, a dummy variable will be added into the model; b and c are the coefficient.

TABLE II SOME REFERENCES ABOUT THE TWO TYPES

Type	Features	First author name	Title	Brief contribution
Type I DZ	Defined in terms of distance to stop line; Usually calculated with constant speed, and acceleration/deceleration rates.	Gazis	The problem of amber signal light in traffic	First put forward the concept and gave the definition
		Sheffi	Model of driver behaviour at high speed signalized intersection	Further defined the concept
		Li keeping	Amber interval design at urban signalized intersections	Calculate the dilemma zone with different yellow light interval and speed
Type II DZ	defined in terms of travel time/distance to stop line; widely used in DZ protection system	Parsonson	Small-area detection at intersection approaches	Proposed the type II dilemma
		Zegeer	Effectiveness of green-extension systems at high-speed intersections	Further defined the type II dilemma
		Li Pengfei	Stochastic dilemma hazard model at high-speed signalized intersections	Build stochastic dilemma hazard model based on type II dilemma zone

III. DILEMMA HAZARD MODEL

A. The Background of Dilemma Hazard Model

Traffic conflicts rather than traffic accidents are often used to evaluate intersection safety because traffic conflicts are much easier to be observed. Zegeer and Deen examined TCs frequency variation before-after installing green extension system [8]. Collecting enough samples of TCs is difficult although the TC techniques have advantages over the actual accident study, so new measurement need to replace it.

The DZ problem is a leading cause for crashes at high-speed signalized intersections, Zimmerman used the number of vehicle trapped in DZ as the surrogate measurement of intersection safety [27]. According to previous study, this kind of measurement is based on a hypothesis that all vehicles in DZ share the same probability to have conflicts and the relationship between crashes and number in DZ isn't clear. Thus, that using number in DZ as the safety surrogate needs to be further verified [18].

B. Dilemma Hazard Model

Li and Abbas proposed a dilemma hazard model, which is based on previous studies on drivers' response to the yellow onset, vehicle kinematics, and a Monte Carlo simulation framework [3]. Li considered the single-vehicle scenario and multiple vehicle scenario, respectively. In single-vehicle situation, only the right-angle collision hazard is expected. The dilemma hazard for the single-vehicle scenario is as follows:

$$HS = P_{cross} * P\{[v_0(y+r) + 0.5a_1(TTI)](y+r-\delta)^2 \leq d+W+L/cross\} + P_{stop} * P\{0.5V_0^2 > a_2(TTI)*d/stop\} \quad (9)$$

where P_{stop} and P_{cross} are stopping and crossing probability, respectively; TTI is the time to intersection at yellow onset; r is all-red clearance duration in seconds; δ is PRT in seconds; d is the distance from the stop bar; W is intersections' width; L is the length of the vehicle.

Multiple vehicle scenario limits study scope to two consecutive vehicles in DZ and it may generate another type of potential hazard, namely, the rear-end collision hazard. The second vehicle is the subject vehicle and the first vehicle is considered not influenced by the subject vehicle. Li and Abbas listed four conditions for subject vehicle that may lead to a dilemma hazard and these four cases are

mutually exclusive. Case 1 supposed that subject vehicle have no lead vehicle, it may be involved in rectangular crash. Case 2 described that both the lead and subject vehicle choose to stop so the subject vehicle may have a potential rear-end hazard. Case 3 sets a condition where the subject vehicle chooses to cross the intersection while the lead vehicle chooses to stop, which is involved in a rear-end crash. Case 4 is like the case 2, where both the two vehicles choose to cross. Then, the case 1 is described in detail.

Case1: the lead vehicle has left the DZ; this situation is the same as the single-vehicle scenario. If the lead vehicle is in DZ, the headway between the lead vehicle and second vehicle is less than $(TTI-DZL)$. The probability can be expressed as

$$P\{headway \leq (TTI-DZL)\} \quad (10)$$

where DZL is the travel time from the near end of DZ to the stop bar and the potential hazard can be expressed as

$$P1 = HS * (1 - P\{headway \leq (TTI-DZL)\}) \quad (11)$$

where H_s can be obtained in (8).

Dilemma hazard model provides a new safety measurement for DZ protection systems and can be used to calculate the optimal clearance (yellow + all - red) interval. However, it would be better to expand to a much wider range of scenarios.

IV. METHODS FOR DZ PROTECTION

According to GHM model, the duration time of yellow light is calculated to minimize DZ. There are some measurements taken into dilemma zone protection and the methods can be divided into four types: resetting the yellow light interval, advance warning system, advance detector system as well as in-vehicle warning system.

A. Resetting the Yellow Light Interval

Resetting the yellow light interval is often seen as the major method to keep vehicles from DZ. According to (1) and (2), DZ can be eliminated when $X_S = X_C$ and the result shows in (3). In (3), the speed is usually considered normally distributed and equals to the 85th percentile speed [24]. Besides, PRT δ and maximum deceleration rate are calculated with constant value. However, the boundary of DZ will change with different driver and vehicle. Thus, there is still a possibility that vehicles will be trapped in DZ even though the yellow intervals have been reset. Obviously, if the yellow

interval is long enough, the number of vehicle trapped in DZ will decrease. In that case, the existence of yellow signal is less meaningful and it will cause other problems. For example, the driver may think it's long enough to cross the intersection, which, as a result, may lead to more running red phenomena. Some researchers have found that no matter how slow the vehicle is, DZ did exist[13].

B. Advance Warning System

Advance warning system provides drivers with warning signal by means of warning signs or warning light. Sayed pointed out that efficient advance warning would diminish the probability that vehicle in DZ[28]. Advance warning flashers is one of the prevailing warning method for DZ protection. It was established near the intersection to warn drivers to prepare for stop. One of the most common configurations is a yellow square sign with two flashing lights and warning like "Prepare to Stop When Flashing". Researches carried out by Sayed show that intersections established with advance warning system had 10% lower overall accident rate than those without it[6]. Some negative effects, such as drivers would be more willing to accelerate at the intersection with advance warning system, turned up undesirably[29]. Drivers couldn't see the advance stop warning until approach the intersection. In that case, they may be caught in rear-end collision due to hard brake.

C. Advance Detector System

Advance detector system plays a role mainly by extending or terminating the green light interval based on information provided by detectors, to a large extent. GES (green extension system) and D-CS are two typical system studied by a lot of researchers. Vehicle detection systems with in-pavement inductive loops could extend the green signal in order to enable a vehicle having crossed the stop line can get through the intersection safely [8]. Besides, modern radar-based smart sensors make it possible to track individual vehicles in close proximity to an intersection. These advancements in technology potentially enable the provision of vehicle and site-specific decision dilemma zone protection at the onset of the yellow indication. D-CS (detection, control and warning system) combines the advance detective system with the advance warning system, which turns out working better

than the former two systems[30] D-CS chooses the optimal time to terminate the green signal by considering both the number of vehicles in DZ and the waiting time of vehicles which is about to obtaining the right to go soon. D-CS calculate five kinds of DZ with the speed of 5th percentile, 15th percentile, 50th percentile, 85th percentile, 90th percentile, while GES calculates the DZ only with the 85th percentile speed. Compared with GES, the boundary calculated by D-CS will more precise.

Even though D-CS considers five speed totally, there are still limitations. The boundaries of DZ are calculated in advance, while the boundary of DZ will dynamically vary with different vehicles. D-CS will lose efficacy once the time to the stop bar at the onset of yellow light beyond the range from 2.5s to 5.5s[7].

D. In-vehicle Warning System

In-vehicle warning system used for DZ protection is based on CVIS (cooperative vehicle infrastructure system). Information of the moving vehicles and the road can be obtained by wireless communication devices. Given that the realization of real-time information on vehicles and roads, in-vehicle warning system could be able to provide the driver with timely visual and auditory warning. In-vehicle system can warn to accelerate or decelerate according to the CVIS and contains three major algorithms: DZE (dilemma zone estimation), DZP (dilemma zone prediction) and WS (warning selection) [31]. DZE and DZP are used to estimate whether the vehicle will be trapped in DZ and WS can help to select the way and information of warning.

In-vehicle warning system has an advantage that it can get real-time information and provide DZ protection for every single vehicle. However, study on this kind of DZ protection is going on and most researches are based on simulation.

V. CONCLUSION

In the first section we have introduce what causes the dilemma zone and then give the definition of the two types of dilemma zone. Dilemma zone hazard is found to be dependent on the vehicle yellow light onset position and approaching speed, which can be used not only to design the yellow interval time to reduce the probability of conflicts but also to analyse the safety

of the intersection. Then, the reason why traffic conflicts appear was explained. The number of vehicles in the dilemma zone is recommended to be the intersection safety surrogate, which lay a basis for the dilemma zone hazard model. To avoid the dilemma zone hazard, masses of researchers are devoted to studying DZ protection.

The methodologies of dilemma zone have developed a lot since 1960. Over five decades, researchers have made great progress in studying factors affecting it and putting the dilemma zone hazard measurement model and methods of DZ protection into practice. However, there still exists some aspects needing to be further studied. As this paper shows, type II DZ is kind of decision problem and the effects of vehicle's type as well as approaching surroundings need to be investigated in next step. What's more, drivers' risk preference is relevance to the probability, too. Some researchers suggested to use perception reaction time to indicate the degree of drivers' aggressiveness. However, it would be better to put demographics into consideration; the effects of latent variables and interactive effect between variables can't be ignored[32].

According to Chiou, drivers approaching the intersections with GSCT(green signal countdown timer) simultaneously decrease crossing probability, create potential for rear-end collision and negative impacts on intersection efficiency[32]. Nevertheless, evidence in other paper is conflicting about the effects of GSCT on type II DZ distribution or intersection capacity. FU believe it might result from varying times of day for observation, data collection methods, sample sizes, analysis approaches, and traffic conditions across different studies. Therefore, the findings on GSCT' effects on type II DZ distribution need to be further verified [33].

As for DZ hazard and protection, scenarios could make a great difference to the DZ hazard model so a modified one thinking about more scenarios might be much closer to truth.

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