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RESEARCH ARTICLE

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DOES THE COUNT OF RADIOACTIVE EVENTS OBEY THE BESSEL DISTRIBUTION?

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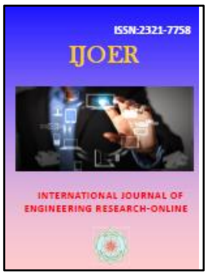

ABSTRACT

We highlight some basic characteristics of the radioactive decay process, with regard to its probabilistic aspect, that is, how many nuclear events occur in a certain time interval, from the disintegration of a radioactive sample.

Based on some examples the statistical character of the radioactivity is presented according to the Poisson distribution. The validity of this distribution is questioned when considering the counting process, since, in the actual data acquisition system are included, incidentally, disturbing effects such as noise and dead time.

Thus the main purpose of this work is to deduce a mathematical expression for the distribution of Probabilities of the Difference between event counts. The distribution of the Differences is justified since this is the procedure by which the real activity of the sample is determined, that is, by the subtraction between the pairs of records associated with the sample and the noise, respectively.

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INTRODUCTION

Radioactive disintegration, when involving a large number of particles, is an example of a physical phenomenon characterized by individual and random processes that provides a well-determined statistical result [(12)]. If N_0 unstable particles are observed at an initial time t_0, it is possible to establish how many will exist at a certain time t. This makes it evident that radioactive particles can be used as a kind of clock since, given their half-life, counting the number of particles that have not decayed, it provides information on the time interval elapsed since the initial instant t_0.

In other words, we can obtain the time by considering the knowledge of the initial and final numbers of the particles, along with the distribution of probabilities.

There are countless radioactivity counting applications in modern physics experiments. Experiments on the possible construction of atomic clocks as well as to test the validity of Bell's inequality, which is an experimental realization of Bohm's proposal to evaluate the EPR paradox, are typical examples. In this first proposition (clocks), to establish the time between consecutive events or time necessary for disintegrations of radioactive substances, it is necessary to count and use a mathematical law

The mathematical models necessary for the study of the disintegration of radioactive atoms developed mainly by VON SCHWINDLER AND BATEMAN[(10)] at the beginning of the 20th century, show that the phenomenon is governed by the Poisson distribution.

Thus, a primary question can be established: From a certain radioactive sample and a counter is it possible for a counting experience to reproduce a numerical series that fits into a Poisson statistic?

In theory, the judgment is affirmative, to the point that it is possible to frame any luminous sign in a Poisson statistic, as BASEIA[2] reports.

However, it is noteworthy that this fact is an open question since that in several moments it is found in the literature that a detailed differentiation is not made between the radioactive decay phenomenon and the nuclear radiation counting mechanism, which are very different.

On the other hand, it is noticeable the concern of some authors, such as Ruark & Devol[10], together with Kinsella et al[5], to highlight the main differences between the distributions associated with each case.

By the way, a series of numbers follows the Poisson distribution, if the probability of occurrence of n events at time t, is given by

$$P_n\left(t/\lambda\right)=\frac{\left(t/\lambda\right)^n}{n!}e^{-t/\lambda} \qquad (1)$$

where λ is the time between two successive events and (t/λ), also represented by E, indicates the average number of disintegration events.

EXPLAINING BETTER

In a time interval of, say, one minute, what is the numerical value indicated by an ideal counting equipment, which monitors the radioactive sample?

It may be any number between zero and infinity. Obviously the greatest probability will be to obtain a numerical value corresponding to the mean value of the sample disintegrations.

If the average value of the nuclear disintegration events is, for example, 10 disintegrations per minute, it is more likely that during a counting time of one minute the recording of a number very close to 10 is obtained in the counting apparatus .

However, even if the mean number of disintegrations per minute equals 10 and the count time one minute, values such as zero, 20, 50, 100, 1000, ... , and so forth may occur, although they are less likely that is, an excited atom may or may not decay at the end of a given time interval.

ON METHODOLOGY IN THE COUNTING EXPERIENCE

Until then, only the events associated with the radioactive sample were mentioned, without considering the counter and the external medium. Radioactive counting devices, on the other hand, exhibit non-ideal characteristics, spurious effects, such as dead counting time, “pile-up” effect among others, which cause fluctuations in counting efficiency[7].

These undesirable effects plus the background radiation also called background, lead to distortions in the original model, generating divergences[5] in the a priori Poisson distribution.

As a result, the data observed during measurements of radioactivity are generally distorted, as has already been observed by some authors, among which is BERKSON[3], who made the following question: "Decay events do they really follow an exponential Poisson function?

The aforementioned author presents the inquiry after examining a series of Am^{241} radioactive decay data and concludes that there is a deviation in the behavior of the data, with respect to the framework in the predicted statistical distribution. This inquiry motivated us to deduce a distribution function for the data arising from the difference between the signal and the noise.

NOISE SUBTRACTION

According to MATVIENKO[8], the procedure to determine the activity of a radioactive sample should follow some steps, among which we highlight:

- Count the sample on the counter device.
- Remove the sample and repeat the count with the “empty” counter to establish the noise level.
- Repeat the previous procedures several times.
- The actual sample activity will then be obtained by subtracting the pairs of numbers associated with sample and noise count.

It is worth mentioning, for the sake of illustration, a technical comment made by renowned researchers in the nuclear field, ASPECT et al.[1] who, during the experimental design of a procedure to test the EPR paradox, write:

“Typical counting rates, greater than $10^4\ s^{-1}$, are high compared to noise rates, which are in the range of $10^2\ s^{-1}$. By subtraction of the accidental rates, approximately $10\ s^{-1}$, of the total rates, one obtains

the true count coincidence rate, in a direction given by a standard vector (a, b), denoted by R_{++}".

Aspect's words prove that the data obtained during the counting of radiation is a set of differences of two measures.

Under this scenario, the suspicion arises that the collection of numbers found does not obey a Poisson probabilistic law, since it is a composition of distributions, usually with two or more parameters.

Moreover, Greenwood and Yule, together with Erlang, cited by HALD[4], proposed composite Poisson distributions, and PAZDUR[9] used them in practical works, especially negative binomial, consisting of the combination of gamma and Poisson functions.

DISTRIBUTION OF THE DIFFERENCE BETWEEN EVENTS

For the purpose of example, a particular case will be presented, which consists in the probability of obtaining a difference of, say, 10 events between sample and noise count. For this we denote the average values for the sample and noise activity, respectively, by E_s and E_b.

The probability of encountering such a difference of 10 events will satisfy one of the following conditions: ten records for sample and zero for noise, or, ten for noise and zero for sample.

However, getting eleven records for sample and one for noise or vice versa is also satisfactory. It is also possible twelve and two, or thirteen and three, and so on.

Considering the addition and multiplication of probability laws, it is possible to write the distribution of ten event differences between sample and noise that will be indicated by D_{10}.

RESULTS AND DISCUSSION

From the exposed conditions we have for D_{10} the expression

$P_{10}(E_s)\ P_0(E_b) + P_0(E_s)\ P_{10}(E_b) + P_{11}(E_s)\ P_1(E_b) + P_{11}(E_b)\ P_1(E_s) + \ldots\ldots$

that can be written as

$$D_{10} = \sum P_{j+10}(E_s)\, P_j(E_b) + P_j(E_s)\, P_{j+10}(E_b) \qquad (2)$$

Generalizing, for a difference of n events we have

$$D_n = \sum P_{j+n}(E_s)\, P_j(E_b) + P_j(E_s)\, P_{j+n}(E_b) \qquad (3)$$

and

$$D_0 = \sum P_j(E_s)\, P_j(E_b)$$

Substituting (1) into (2), and considering the Bessel functions

$$I_n(x) = \sum_{j=0}^{\infty} \frac{\left(x/2\right)^{n+2j}}{(n+j)!\,j!} \qquad (4)$$

we get

$$D_n = \left(h^n + h^{-n}\right) e^{-(E_s+E_b)} I_n(x) \quad (n \neq 0) \qquad (5)$$

with

$$D_0 = e^{-(E_s+E_b)} I_0(x) \quad (n=0)$$

where

$$h = \sqrt{\frac{E_s}{E_b}} = \sqrt{\frac{\lambda_b}{\lambda_s}} \qquad x = 2\sqrt{E_s E_b}$$

If $\lambda_b >> \lambda_s$ then

$$D_n \cong h^n e^{-t/\lambda_s} I_n(x) \qquad (6)$$

$$D_0 \cong e^{-t/\lambda_s} I_0(x)$$

NORMALIZATION OF THE FUNCTION D_n

By definition, the norm of the functions Dn, is given by:

$$N(D_n) = \sum_{n=0}^{\infty} e^{-(E_b+E_s)}\left(h^n + h^{-n}\right) I_n(x) \qquad (7)$$

Using the function that generates the Bessel functions we have:

$$\sum_{n=0}^{\infty}\left(h^n + h^{-n}\right) J_n(x) = e^{\frac{x}{2}\left(h - \frac{1}{h}\right)} \qquad (8)$$

Replacing $I_n(x) = i^{-n} J_n(ix)$ in (7) results in

$N(D_n)=1$

AVERAGE VALUE OF DISTRIBUTION OF DIFFERENCES

The mean value of the mentioned function is defined by

$$< n >_{D_n} = \sum_{n=0}^{\infty} n D_n = e^{-(E_s+E_B)} \sum_{n=0}^{\infty} n\left(h^n + h^{-n}\right) I_n(x) \qquad (9)$$

and deriving the generative function of Bessel functions with respect to h, results in

$$\sum_{n=0}^{\infty}\left(n h^{n-1} + n h^{-n-1}\right) J_n(x) = \frac{x}{2}\left(1 + \frac{1}{h^2}\right) e^{\frac{x}{2}\left(h - 1/h\right)} \qquad (10)$$

which replaced in (9) produces

$$< n >_{D_n} = E_s - E_b$$

CONCLUSION

In the analysis of the issue in question, it was shown that the Distribution of Differences is not of the Poisson type, but presents an exponential dependence modulated by a Bessel function, more comprehensive than that of a simple nuclear disintegration mainly due to the dependence of two-parameter, E_s and E_b, associated with sample and noise.

It is therefore a composite distribution consisting of two variables in the style proposed by Erlang and Greenwood-Yule, applied by PAZDUR(9). Then, the event counting analyzes should consider a more general distribution, with sample parameters and spurious effects, since the Poisson distribution is not valid when subtracting events.

Thus, only the precise knowledge of the distribution function along with a well-defined counting process is that it becomes possible to know, with precision, the time, hypothetically, desired.

Particularly, in relation to the conclusion of the results analyzed by Aspect(1) for the statement "Uncertainty is the standard deviation, which corresponds to that defined by the Poisson law", a certain precaution is necessary in order to properly interpret the mathematics of the numbers obtained.

Thus, in order to identify and establish the deviations that, in general, are not related to the Poisson law, it is necessary to study theoretically, under different aspects, the process of disintegration and counting of the radioactivity.

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