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ANTI-FUZZY STRONG BI-IDEALS OF NEAR-RINGS

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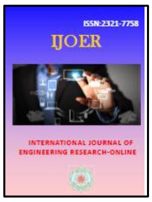

ABSTRACT
In this paper we introduce the notation of anti-fuzzy strong bi-ideal of a near-ring. We have discussed some of their theoretical properties in detail and obtain some characterization.

Key words: Anti-Fuzzy two sided N-subgroup, Anti-fuzzy subnear-ring, Anti-fuzzy bi-ideal, Anti-fuzzy strong bi-ideal

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1. INTRODUCTION

Zadeh [10] introduced the concept of sets in 1995. The notions of anti-fuzzy ideal and anti-fuzzy N-subgroup of a near-ring were introduced by Kim, Jun and Yon [3,4]. In this paper, we introduce the notion of a anti-fuzzy strong bi-ideal of a near-ring We establish that every anti-fuzzy left N-subgroup or anti-fuzzy left ideal of a near-ring is a anti-fuzzy strong bi-ideal of a near-ring and also we establish that every left permutable fuzzy right N-subgroup or left permutable anti-fuzzy right ideal of a near-ring is a anti-fuzzy strong bi-ideal of a near-ring. But the converse is not necessarily true as shown by an example. Further, we discuss the properties of anti-fuzzy strong bi-ideal of a near-ring and provide example. Throughout this paper N will denote a right near-ring unless otherwise specified.

2. Preliminaries

Definition: 2.1

A non empty set N together with two binary operations "+" and "·" is called be a **near-ring** [1] if it satisfies the following axioms:

(i)(N,+) is a group.

(ii)(N,·) is a semi group.

(iii)$(x + y)\cdot z=(x\cdot z)+y\cdot z$, for every $x,y,z\in N$.

Note: 2.2

(i) Let X be a near-ring. Given two subsets A and B of X, $AB = \{ab/a\in A, b\in B\}$. Also we define another operation "$*$" $A*B = \{a(b+i)-ab/a, b\in A, i\in B\}$.

(ii) $0x = 0$. In general $x0 \neq 0$, for some x in N.

Definition: 2.3

A near-ring N is called **zero-symmetric**, if $x0 = 0$, for all x in N.

Definition: 2.4

A subgroup A of (N,+) is called a **bi-ideal** of near-ring N if $ANA\cap(AN)*A\subseteq A$.

Definition: 2.5

An element $a\in N$ is said to be **regular** if for each $a\in N$, $a = aba$, for some $b\in N$.

Definition: 2.6

A near-ring N is said to be **left permutable** near-ring if $abc = acb$, for all a,b,c in N.

Definition: 2.7

A function A from a non-empty set X to the unit interval [0,1] is called a fuzzy subset of N [14].

Notation: 2.8

Let A and B be two fuzzy subsets of N. We define the relation $\subseteq$ between A and B, the union, intersection and anti product of A and B, respectively as follows:

(i) $A\subseteq B$ if $A(x)\leq B(x)$, for all $x\in N$,

(ii) $(A\cup B)(x) = \max\{A(x), B(x)\}$, for all $x\in N$,

(ii) $(A\cap B)(x) = \min\{A(x), B(x)\}$, for all $x\in N$,

(iii) $(A \circ_a B)(x) = \begin{cases} \inf\limits_{x=yz}\{\max\{A(y), B(z)\}\} & \text{if } x = yz, \text{ for all } y, z\in N, \\ 0 & \text{otherwise} \end{cases}$

We denote by the anti characteristic function of N is denoted by **N**, that is, **N**(x) = 0, for all $x\in N$.

Definition: 2.9

A fuzzy subset A of a group (N,+) is said to be an anti fuzzy subgroup of N if for all $x,y\in N$,

(i) $A(x+y) \le \max\{ A(x) , A(y) \}$

(ii) $A(-x) = A(x)$,

Or equivalently $A(x - y) \le \max\{ A(x) , A(y)\}$.

Note: 2.10

If A is an anti fuzzy subgroup of a group (N,+), then $A(0) \le A(x)$, for all $x\in N$.

Definition: 2.11

A fuzzy subset A of N is called an **anti fuzzy subnear-ring** of N if for all $x,y\in N$,

(i) $A(x - y) \le \max\{A(x) , A(y)\}$

(ii) $A(xy) = \max\{A(x) , A(y)\}$

Definition: 2.12

A fuzzy subset A of N is said to be an **anti fuzzy two-sided N-subgroup** of N if

(i) A is an anti fuzzy subgroup of (N,+),

(ii) $A(xy) \le A(x)$, for all $x,y\in N$,

(iii) $A(xy) \le A(y)$, for all $x,y\in N$.

If A satisfies (i) and (ii), then A is called an anti fuzzy right N-subgroup of N. If A satisfies (i) and (iii), then A is called an anti fuzzy left N-subgroup of N.

Definition: 2.13

A fuzzy subset A of N is said to be an **anti fuzzy ideal** of N if

(i) A is an anti fuzzy subnear-ring of N,

(ii) $A(y+x-y) = A(x)$, for all $x, y\in N$,

(iii) $A(xy) \le A(x)$, for all $x, y\in N$,

(iv) $A(a(b+i)-ab) \le A(i)$, for all $a, b, i, \in N$.

If A satisfies (i) and (ii) and (iii) then A is called an anti fuzzy right ideal of N. If A satisfies (i), (ii) and (iv), then A is called an anti fuzzy left ideal of N. In case of zero-symmetric, If A satisfies (i), (ii) and $A(xy) \le A(y)$, for all $x, y\in N$ and A is called an anti fuzzy left ideal of N.

Definition: 2.14

An anti fuzzy subgroup A of N is called an anti fuzzy bi-ideal of N if $(A\circ_a N \circ_a A)\cup((A\circ_a N) *_a A) \supseteq A$. In case of zero-symmetric, An anti fuzzy subgroup A of N is called an anti fuzzy bi-ideal of N if $A\circ_a N \circ_a A \supseteq A$.

3. Anti Fuzzy Strong Bi-ideals of Near-Rings

Definition: 3.1

An anti fuzzy bi-ideal A of N is called an **anti fuzzy strong bi-ideal** of N, if

$\mathbf{N}\circ_a A\circ_a A \supseteq A$.

Example: 3.1.1

Let N={0,a,b,c} be a near-ring with two binary operations '+' and '·' is defined as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	a	c	b
b	0	b	b	0
c	0	c	a	b

Define a fuzzy subset $A : N\to[0,1]$ by A(0) = 0.2, A(a) = A(b) = 0.7, A(c) = 0.3.

Then $(A\circ_a N\circ_a A)(0) = 0.7$, $(A\circ_a N \circ_a A)(a) = 0.7$, $(A\circ_a N\circ_a A)(b) = 0.7$, $(A\circ_a N \circ_a A)(c) = 0.7$,

$(\mathbf{N}\circ_a A\circ_a A)(0) = 0.7$, $(\mathbf{N}\circ_a A\circ_a A)(a) = 0.7$, $(\mathbf{N}\circ_a A\circ_a A)(b) = 0.7$, $(\mathbf{N}\circ_a A\circ_a A)(c) = 0.7$, and so A is an anti fuzzy strong bi-ideal of N.

Note: 3.2

Every anti fuzzy strong bi-ideal is an anti fuzzy bi-ideal. But the converse is not true.

Example: 3.2.1

Let N={0,a,b,c} be a near-ring with two binary operations '+' and '·' is defined as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	0	b	0
c	0	0	c	0

Define a fuzzy subset $A : N\to[0,1]$ by A(0) = 0.2, A(a) = A(b) = 0.7, A(c) = 0.9. Then

$(A\circ_a N\circ_a A)(0) = 0.7$, $(A\circ_a N \circ_a A)(a) = 0.7$, $(A\circ_a N\circ_a A)(b) = 0.7$, $(A\circ_a N \circ_a A)(c) = 0.9$, $(\mathbf{N} \circ_a A\circ_a A)(0) = 0.7$, $(\mathbf{N} \circ_a A\circ_a A)(a) = 0.7$, $(\mathbf{N} \circ_a A\circ_a A)(b) = 0.7$, $(\mathbf{N} \circ_a A\circ_a A)(c) = 0.7$. Then A is an anti fuzzy bi-ideal of N. But not an anti fuzzy strong bi-ideal, since $(\mathbf{N} \circ_a A\circ_a A)(c) \not\supseteq \mathbf{A}(c)$.

Theorem: 3.3

Let $\{A_i : i\in J\}$ be any family of anti fuzzy strong bi-ideals N. Then $A = \bigcup_{i\in J} A_i$ is an anti fuzzy strong bi-ideal of N, where J be an index set.

Proof:

Let $\{A_i : i\in J\}$ be any family of anti fuzzy strong bi-ideals of N. Now for all $x,y\in N$,

$\underset{i\in J}{\cup}A_i(x-y) = \max\{A_i(x-y) / i\in J\} \le \max\{\max\{A_i(x), A_i(y)\} / i\in J\}$(Since A_i is an anti fuzzy subgroup of N)

$= \max\{A_i(x), A_i(y) / i\in J\}$

Therefore A is an anti fuzzy subgroup of N.

Now for all $x\in N$, since $A = \underset{i\in J}{\cup}A_i \supseteq A_i$, for every $i\in J$

$(A\circ_a N \circ_a A)\cup((A\circ_a N) *_a A)(x) \ge (A_i\circ_a N \circ_a A_i)\cup((A_i\circ_a N) *_a A_i))(x)$(Since A_i is an anti fuzzy bi-ideal of N)

$\ge (A_i)(x)$, for all $i\in J$

It follows that,

$(A\circ_a N \circ_a A)\cup((A\circ_a N) *_a A))(x) \ge \sup\{A_i(x) : i\in J\} = (\underset{i\in J}{\cup}A_i)(x) = A(x)$

Thus $(A\circ_a N \circ_a A)\cup((A\circ_a N) *_a A) \supseteq A$. So A is an anti fuzzy bi-ideal of N.

$(\mathbf{N} \circ_a A\circ_a A)(x) \ge (\mathbf{N} \circ_a A_i\circ_a A_i)(x) \ge A_i(x)$ for every $i\in J$

(since A_i is an anti fuzzy strong bi-ideal of N)

It follows that, $(\mathbf{N} \circ_a A\circ_a A)(x) \ge \sup\{A_i(x) : i\in J\}$

$= (\underset{i\in J}{\cup}A_i)(x) = A(x)$

Thus $\mathbf{N} \circ_a A\circ_a A \supseteq A$. So A is an anti fuzzy strong bi-ideal of N.

Theorem: 3.5

Every left permutable anti fuzzy right N-subgroup of N is a anti fuzzy strong bi-ideal of N.

Proof:

Let A be a left permutable anti fuzzy right N-subgroup of N. To prove A is an anti fuzzy strong bi-ideal of N.

Choose a, b, c, x, y, i, b_1, b_2, x_1, x_2, y_1, y_2 in N such that $a = bc = x(y+i)-xy$, $b = b_1 b_2$, $x = x_1 x_2$ and $y = y_1y_2$. Then

$(A\circ_a N \circ_a A)\cup((A\circ_a N) *_a A)(a) = \max\{(A\circ_a N \circ_a A)(a),((A\circ_a N) *_a A)(a)\}$

$= \max\{\inf_{a=bc} \max(A\circ_a \mathbf{N})(b),A(c)\},((A\circ_a N) *_a A)(x(y+i)-xy)\}=$

$\max\{\inf_{a=bc}\max\{\inf_{b=b_1b_2} \max\{A(b_1),\mathbf{N}(b_2)\}, A(c)\},(A\circ_a N) *_a A)(x(y+i)-xy)\}$

(since $\mathbf{N}(z) = 0$, for all $z\in N$)

$=\max\{\inf_{a=bc}\max\{\inf_{b=b_1b_2}\{A(b_1), A(c)\},(A\circ_a N)*_a A)(x(y+i)-xy)\}$

(Since A is a anti fuzzy right N-subgroup of N, $A(bc) = A((b_1b_2)c) = A(b_1(b_2c)) \le A(b_1)$

$\ge\max\{\inf_{a=bc}\max\{A(bc),\mathbf{N}(c)\},\mathbf{N}(x(y+i)-xy)\}$

$= \max\{\inf_{a=bc}\max\{A(bc),\mathbf{N}(x(y+i)-xy)\} = A(bc) = A(a)$

Thus $(A\circ_a N \circ_a A)\cup((A\circ_a N) *_a A)\supseteq A$. Hence A is a anti fuzzy bi-ideal of N.

Choose a, b, c, $b_1, b_2\in N$ such that $a = bc$ and $b = b_1b_2$. Then

$(\mathbf{N} \circ_a A\circ_a A)(a) = \inf_{a=bc}\max(\mathbf{N} \circ_a A)(b),A(c)\}$

$= \inf_{a=bc}\max\{\inf_{b=b_1b_2} \max\{\mathbf{N}(b_1), A(b_2)\}, A(c)\}$

$= \inf_{a=bc}\max\{\inf_{b=b_1b_2}\{A(b_2), A(c)\}$

(Since A is a left permutable anti fuzzy right N-subgroup of N, $A(bc) = A((b_1b_2)c) = A((b_2b_1)c) \le A(b_2) \ge \inf_{a=bc}\max\{A(bc), \mathbf{N}(c)\}$

$= \inf_{a=bc}\max\{A(bc), 0\}$

$= \inf_{a=bc}A(bc) = A(a)$

Therefore $\mathbf{N}\circ_a A\circ_a A\supseteq A$. Hence A is an anti fuzzy strong bi-ideal of N.

Theorem: 3.6

Every anti fuzzy left N-subgroup of N is an anti fuzzy strong bi-ideal of N.

Proof:

Let A be an anti fuzzy left N-subgroup of N. To prove A is a anti fuzzy strong bi-ideal of N.

Choose a, b, c, x, y, i, c_1, c_2, x_1, x_2, y_1, y_2 in N such that $a = bc = x(y+i)-xy$, $c = c_1,c_2$, $x = x_1 x_2$ and $y = y_1y_2$. Then

$(A\circ_a N \circ_a A)\cup((A\circ_a N) *_a A))(a) = \max\{(A\circ_a N \circ_a A)(a),((A\circ_a N) *_a A))(a)\}$

$= \max\{\inf_{a=bc}\max\{(A(b),\mathbf{N}\circ_a A(c)\},((A\circ_a N) *_a A)(x(y+i)-xy)\}$

$= \max\{\inf_{a=bc}\max\{A(b), \inf_{c=c_1c_2}\max\{\mathbf{N}(c_1), A(c_2)\}\},((A\circ_a N) *_a A)(x(y+i)-xy)\}$

$=\max\{\inf_{a=bc}\max\{A(b), \inf_{c=c_1c_2}A(c_2)\},(A\circ_a N)*_a A)(x(y+i)-xy)\}$

(Since A is a anti fuzzy left N-subgroup of N, $A(bc) = A(b(c_1c_2)) = A(bc_1)c_2) \le A(c_2)$

$\ge\max\{\inf_{a=bc}\max\{\mathbf{N}(b), A(bc)\},\mathbf{N}(x(y+i)-xy)\} = A(bc) = A(a)$

Thus $(A\circ_a N \circ_a A)\cap((A\circ_a N) *_a A)\supseteq A$. Hence A is an anti fuzzy bi-ideal of N.

Choose a, b,c, $c_1, c_2\in N$ such that $a = bc$ and $c = c_1, c_2$. Then

$(\mathbf{N} \circ_a A\circ_a A)(a) = \inf_{a=bc}\max\{(A(b),(\mathbf{N} \circ_a A)(c)\}$

$= \inf_{a=bc}\max\{A(b), \inf_{c=c_1c_2}\max\{\mathbf{N}(c_1), A(c_2)\}$

$= \inf_{a=bc}\max\{A(b), A(c_2)\}$

(Since A is an anti fuzzy left N-subgroup of N, $A(bc) = A(b(c_1c_2)) = A((bc_1)c_2) \le A(c_2))$

$\ge \inf_{a=bc}\max\{\mathbf{N}(b), A(bc)\}$

$= \inf_{a=bc}\max\{0, A(bc)\}$

= A(bc) = A(a)

Therefore $\mathbf{N} \circ_a A \circ_a A \supseteq A$. Hence A is an anti fuzzy strong bi-ideal of N.

Theorem: 3.7

Every left permutable anti fuzzy two-sided N-subgroup of N is an anti fuzzy strong bi-ideal of N.

Proof:

The proof is straight forward from the above Theorem 3.5 and Theorem 3.6.

Theorem: 3.8

Every left permutable anti fuzzy right ideal of N is an anti fuzzy strong bi-ideal of N.

Proof:

The proof is similar to that of Theorem 3.5.

Theorem: 3.9

Every anti fuzzy left ideal of N is an anti fuzzy strong bi-ideal of N.

Proof:

Let A be an anti fuzzy left ideal of N. To prove A is an anti fuzzy strong bi-ideal of N.

Choose a, b, c, x, y, i, b_1, b_2, x_1, x_2, y_1, y_2 in N such that a = bc = x(y+i)-xy, $b = b_1 b_2$, $x = x_1 x_2$ and $y = y_1 y_2$. Then

$(A \circ_a N \circ_a A) \cup ((A \circ_a N) *_a A))(a) = \max\{(A \circ_a N \circ_a A)(a), ((A \circ_a N) *_a A))(a)\}$

$= \max\{\inf_{a=bc} \max(A \circ_a \mathbf{N})(b), A(c)\}, (A \circ_a N) *_a A)(x(y+i)-xy)\}$

$= \max\{\{\inf_{a=bc} \max(A \circ_a \mathbf{N})(b_1 b_2), A(c)\}, \inf_{a=x(y+i)-xy} \max\{(A \circ_a \mathbf{N})(x), (A \circ_a \mathbf{N})(y), A(i)\}\}$

(since $A \circ_a \mathbf{N} \supseteq \mathbf{N}$ and since A is a anti fuzzy left ideal of N, $A(x(y+i)-xy) \leq A(i)$)

$\geq \max\{\inf_{a=bc} \max\{\mathbf{N}(b_1 b_2), \mathbf{N}(c)\}, \inf_{a=x(y+i)-xy} \max\{\mathbf{N}(x), \mathbf{N}(y), A(x(y+i)-xy)\}\} = A(x(y+i)-xy) = A(a)$.

Therefore $(A \circ_a N \circ_a A) \cup ((A \circ_a N) *_a A) \supseteq A$. Hence A is an anti fuzzy bi-ideal of N.

Choose a, b, c, $b_1, b_2 \in$ N such that a = bc and $b = b_1, b_2$. Then

$\mathbf{N} \circ_a A \circ_a A(a) = \inf_{a=bc} \max(\mathbf{N} \circ_a A)(b), A(c)\}$

$= \inf_{a=bc} \max\{\inf_{b=b_1 b_2} \max\{\mathbf{N}(b_1), A(b_2)\}, A(c)\}$

$= \inf_{a=bc} \max\{\inf_{b=b_1 b_2} \{A(b_2), A(c)\}$

(Since A is an anti fuzzy left ideal of N, A(a) = A(bc) = A(b(n+c)-bn) ≤ A(c))

$\geq \inf_{a=bc} \max\{\mathbf{N}(b_2), A(b(n+c) - bn)\}$

$= \inf_{a=bc} \max\{0, A(bc)\}$

= A(bc) = A(a)

Therefore $\mathbf{N} \circ_a A \circ_a A \supseteq A$. Hence A is an anti fuzzy strong bi-ideal of N.

Theorem: 3.10

Every left permutable anti fuzzy ideal of N is an anti fuzzy strong bi-ideal of N.

Proof:

The proof is straight forward from 3.8 and Theorem 3.9.

Remark: 3.11

The converse of Theorem 3.7 and Theorem 3.10 are not necessarily true as shown by the following example.

Example: 3.11.1

In example 3.1.1, A is an anti fuzzy strong bi-ideal of N. Since A(a) = A(cb) $\not\leq A(c)$ and A(b) = A(ac) $\not\leq A(c)$, A is not an anti fuzzy two-sided N-subgroup of N. Since A(b) =

A(cc) $\not\leq$ max {A(c),A(c)}, A is not an anti fuzzy sub near-ring of N and so A is not an anti fuzzy ideal of N.

Theorem: 3.12

Let A be any anti fuzzy strong bi-ideal of a near-ring N. Then $A(axy) \leq \max\{A(x), A(y)\}\ \forall\ a, x, y \in N$.

Proof:

Assume that A is an anti fuzzy strong bi-ideal of N. Then $\mathbf{N} \circ_a A \circ_a A \supseteq A$.

Let a, x and y be any element of N. Then

$A(axy) \leq (\mathbf{N} \circ_a A \circ_a A)(axy)$

$= \inf_{axy=pq} \max(\mathbf{N} \circ_a A)(p), A(q)\}$

$\leq \max(\mathbf{N} \circ_a A)(ax), A(y)\} =$

$\max\{\inf_{ax=z_1 z_2} \max\{\mathbf{N}(z_1), A(z_2)\}, A(y)\}$

$\leq \max\{\max\{\mathbf{N}(a), A(x)\}, A(y)\} = \max\{\max\{0, A(x), A(y)\}$

$= \max\{A(x), A(y)\}$

This shows that $A(axy) \leq \max\{A(x), A(y)\}\ \forall a, x, y \in N$.

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