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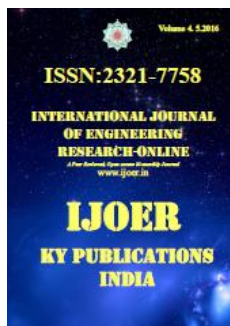
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## RAYBMAN METHOD FOR CORRELATION SYSTEM IDENTIFICATION

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### ABSTRACT

Correlation identification methods have a significant interest in control and system identification engineering; one of these methods is Raybman model-table identification method. This method based on Wiener–Hopf integral equation and allows to make an approximate for structural and parametric identification of stable linear dynamic control objects up to the third order. This paper is an introduction for the Raybman model-table identification method.

**Keywords:** system identification; correlation identification; Raybman model-table method.

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### 1. INTRODUCTION

Dynamic systems are encountered almost everywhere in reality, e.g., in target tracking, chemical reaction, satellite guidance and navigation, power transmission and distribution, orbit determination, weather and financial forecasting, optimal control, fault diagnosis, etc. A continuous increase in the complexity, efficiency, and reliability of modern industrial systems necessitates a continuous development in the control and fault diagnosis theory and practice. An early detection and maintenance of faults can help avoid system shutdown, breakdowns and even catastrophes involving human fatalities and material damage.

The temporal behavior of technical systems from the areas of electrical engineering, mechanical engineering, and process engineering, as well as non-technical systems from areas as diverse as biology, medicine, chemistry, physics, economics, can uniformly be described by mathematical models. This is covered by systems theory. However, the application of systems theory requires that the

mathematical models for the static and dynamic behavior of the systems and their elements are known. The derivation of mathematical system and process models and the representation of their temporal behavior based on measured signals is termed system identification.

Despite the fact that the theoretical analysis can in principle deliver more information about the system, provided that the internal behavior is known and can be described mathematically, system identification has found ever increasing attention. The main reasons that make system identification is preferred over theoretical analysis are a) theoretical analysis can become quite complex even for simple systems. Also some systems are very complex, making the theoretical analysis too time-consuming. b) mostly, model coefficients derived from the theoretical considerations are not precise enough. c) not all actions taking place inside the system are known. And sometimes, the actions taking place cannot be described mathematically with the required

accuracy. and d) identified models can be obtained in shorter time with less effort compared to theoretical modeling.

System identification deals with constructing mathematical models of dynamical systems from measured data. Such models have important applications in many technical and nontechnical areas. System Identification is the process of finding a model that best produces a data, obtained by a system with a known input. Obtaining a model of a system is quite useful in studying its behaviour. This model can be obtained by either the use of physical laws that govern the system or by identification procedures (which can be performed by processing input/output data obtained by performing various experiments). Once a good model is obtained, it can be used for the analysis of the system properties, prediction and controller design. Mathematical models of dynamical systems are of rapidly increasing importance in engineering and today all designs are more or less based on mathematical models. Models are also extensively used in other, nontechnical areas such as biology, ecology, and economy. If the physical laws governing the behavior of the system are known we can use these to construct so called white-box models of the system. In a white-box model, all parameters and variables can be interpreted in terms of physical entities and all constants are known a priori. At the other end of the modeling scale we have so called black-box modeling or identification. Black-box models are constructed from data using no physical insight whatsoever and the model parameters are simply knobs that can be turned to optimize the model fit. Ljung [1] stated a basic rule in estimation, that is, one should not estimate what is already known. In other words, one should utilize prior knowledge and physical insight about the system when selecting the model structure and type to be used. There are three types of models based on the prior knowledge [2] :-

**White Box Models:** This is the case when a model is perfectly known and has been possible to construct it from prior knowledge and physical

insight by taking into account the connection between the components of the system.

**Grey Box Models:** This is the case when some physical insight is available, but several parameters remain to be determined from observed data. It is useful to consider two sub-cases:

**Physical Modeling:** A model structure can be built on physical grounds, which has a certain number of parameters to be estimated from data. This could for example be a state space model of given order and structure.

**Semi-Physical Modeling:** Physical insight is used to suggest certain non-linear combinations of measured data signal. These new signals are then subjected to structures of black box character.

**Black Box Models:** No physical insight is available or used, but the chosen model structure belongs to families that are known to have good flexibility and have been successful in the past. The only chance to get the model is to collect data and use them to guess the links between inputs and outputs.

Despite the quite simplistic nature of many black-box models, they are frequently very efficient for modeling dynamical systems, require less engineering time to construct, and also useful to deal with very complex systems than white-box models [3].

According to the modeling approach, dynamical systems models are generally of two types, parametric non-parametric models. In parametric model, model parameters have meaningful values in the real process (i.e. theoretical model). While in non-parametric model, model parameters have no meaningful values for the process (empirical model, neural network model, etc.). According to this there are two types of identification categories– parametric and non–parametric approaches.

1. **Parametric approach–** suppose a set of candidate models has been selected (such as transfer function or impulse response), and it is parameterized as a model structure using a parameter vector  $\theta$ . The parametric methods determine or estimate the best values of the

vector  $\theta$ . These methods are sometimes referred as parameter estimation methods [1]. Example of the parametric approaches: least square, recursive least square, generalized least square, maximum likelihood and instrumental variables. The parametric approach can take a known model structure or a limited number of unknown parameters.

2. Non-Parametric approach- these methods aim to determine the system model without first selecting a confirmed set of possible models. These methods are often called nonparametric since they do not employ a finite-dimensional parameter vector in the search for a best description [1]. They can be divided into nonparametric time-domain modelling and nonparametric frequency-domain modelling. Example of this approach includes transient analysis, frequency analysis, correlation analysis, spectrum analysis, genetic algorithms (GA) and Neural Networks (NNs). Nonparametric approach may be no definite model structure or described in many points (frequency characteristics, impulse response

In general non-parametric methods are easy to use but they give moderately accurate models. According to that, system identification problem consists of two subtasks:-

- a. Structural identification of the equations in the model,
- b. Parameter identification of the model's parameters  $\tilde{\theta}$ .

**2. Raybman model-table identification method**

Consider a linear stationary dynamic system with one input and one output, described by the convolution integral over the impulse response function based on noise measurement output

$$y(t) = \int_0^{\infty} w(\tau)u(t - \tau)d\tau + \eta(t) \quad 1$$

The problem of identification is to build the impulse response function estimation  $w(t)$  according to the observations of  $y(t)$  and  $u(t)$  at a certain time

interval  $[0, T]$  as shown in figure 1. The average quadratic loss function is

$$Q = \frac{1}{T} \int_0^T [y(t) - y_M(t)]^2 dt = \frac{1}{T} \int_0^T [y(t) - \int_0^{\infty} w(\tau)u(t - \tau)d\tau]^2 dt \quad 2$$

In numerous studies [4,5, 6,7, 8, 9, 11, 12] for the identification of linear dynamic systems shows that the evaluation of the impulse response function linear stationary system obtained by the criterion of minimum mean squared error, defined as the solution of the integral equations of Wiener-Hopf:

$$R_{yu}(t) = \int_0^{\infty} w(\tau)R_{uu}(t - \tau)d\tau \quad 3$$

where  $R_{uu}(t)$  is the autocorrelation function of a stationary ergodic process  $u(t)$ ;  $R_{yu}(t)$ - The cross-correlation function of a stationary ergodic process  $u(t)$  and  $y(t)$ , calculated on the interval  $[0, T]$  for  $T \rightarrow \infty$ . Unfortunately, the problem of finding the impulse response function  $w(t)$  of integral Wiener-Hopf equation (3), except for the special case when the input is absolutely random signal such as white noise, belongs to a class of so-called incorrect problems [4, 6, 7, 10]. In practical terms, the incorrectness of the problem means that even small errors in input data can lead to large errors in the solution of the problem or in general to the inability to obtain any decision. Thus, for the solution of (3) it is necessary to provide the correlation functions in analytical form, which is not always possible, because in practice these functions are determined by numerical methods, the experimentally removed implementations of input-output signals in normal operation of the facility. Furthermore, it is theoretically required to determine the correlation functions for infinite samples, in practice it is possible their estimation methods of mathematical statistics only for a finite time interval  $[0, T]$ . All this leads to random errors in the estimates of correlation functions, which makes the task of solving the Wiener-Hopf equation (3) incorrect problem. Under the guidance of the Russian professor Raybman [7, 13] developed a method called the model table of identification of linear dynamic control object, which avoids many of

the problems associated with the decision incorrect problems when used for the identification of Wiener-Hopf equation (3).

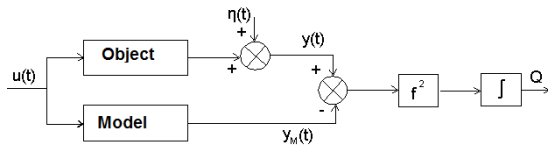


Fig. 1: correlation identification model

### 3. Raybman model-table identification method

The basic idea of this method in identification the impulse transfer function is by calculation the experimental correlation functions  $R_{yu}(t)$  and  $R_{uu}(t)$  and special prepared table [7]. This table contains three columns:- the first one contains the graphic and analytical characteristics of the input action, namely the autocorrelation function  $R_{uu}(t)$ . The second one contains the graphic and analytical characteristics of the output action, namely, cross-correlation function  $R_{yu}(t)$ . The third one contains differential equation objects with these parameters that convert  $R_{uu}(t)$  into  $R_{yu}(t)$ . Identification is produced by the visual selection of the corresponding tabular and

experimental autocorrelation and cross-correlation functions, followed by recalculation of the parameters of the resulting differential equations using graphs found on scaling factor of the correlation functions. This method allows the operator to determine the approximate object with a very low cost of work and time of implementations of input-output signals control object received in its normal mode of operation. However, because of its proximity, it can only be used as a rapid method for the preliminary analysis and solution of problems of identification that do not require very high precision or when the designed control system will be quite rough (insensitive) to the parameters of control object. In addition, this method is completely manual, as required by the experimenter's visual selection of the most similar to the experimental table of auto and cross-correlation function of a sufficiently large number of functions listed in the tables of the book [7]. This method allows make an approximate structural and parametric identification of linear dynamic stable control object to the third order.

Tables below contains samples list of most real systems as written in the Raybman book.

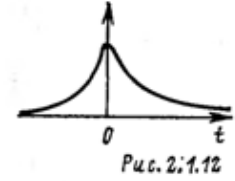
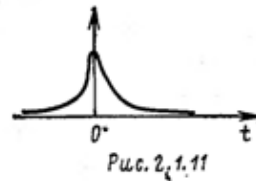
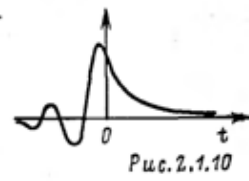
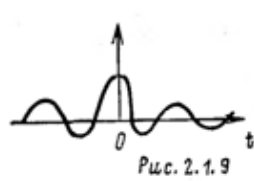
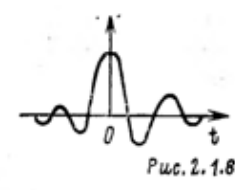
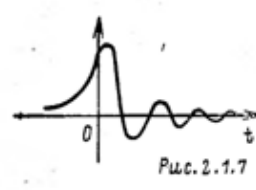
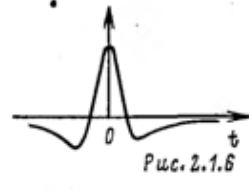
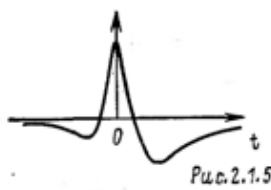
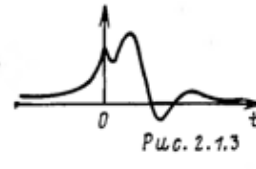
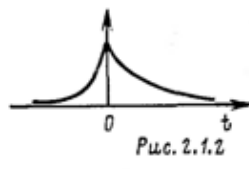
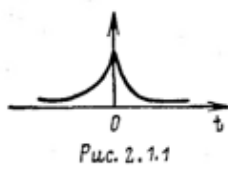
Корреляционная функция входа  $r_{xx}(t) = Ae^{-\alpha|t|}$ ,  $A > 0$ ,  $\alpha > 0$  (рис. 2.1.1)

Важнейшая корреляционная функция выхода и входа	Импульсная характеристика системы	Условия существования решения и условия устойчивости	Передаточная функция системы (при условии существования решения)
1	2	3	4
$r_{yx}(t) = \begin{cases} Be^{-\beta t}, & t \geq 0, \beta > 0; \\ Be^{\alpha t}, & t < 0 \end{cases}$ <p>(рис. 2.1.2)</p>	$g(\tau) = \frac{B}{A} \frac{\alpha + \beta}{2\alpha} [\delta(\tau) + (\alpha - \beta) e^{-\beta\tau}]$	$A > 0;$ $\alpha, \beta > 0;$ $-\infty < B < +\infty$	$G(p) = \frac{B}{A} \frac{\alpha + \beta}{2\alpha} \frac{p + \alpha}{p + \beta}$
$r_{yx}(t) = \begin{cases} \sum_{k=0}^n B_k t^k e^{-\beta t}, & t \geq 0, \beta > 0; \\ B_0 e^{\alpha t}, & t < 0 \end{cases}$ <p>(рис. 2.1.3)</p>	$g(\tau) = \frac{1}{2\alpha A} [(\alpha + \beta) B_0 - B_1] \delta(\tau) - \frac{1}{2\alpha A} e^{-\beta\tau} \sum_{k=1}^{n+1} [(\beta^2 - \alpha^2) B_{k-1} - 2\beta k B_k + k(k+1) B_{k+1}] \tau^{k-1}$	$A > 0;$ $\alpha, \beta > 0;$ $-\infty < B_k < +\infty,$ $k = 0, \dots, n$	$G(p) = \frac{1}{2\alpha A} \frac{p + \alpha}{p + \beta} \left[ B_0 (\alpha + \beta) - \frac{(p - \alpha) \sum_{k=1}^n k! B_k (p + \beta)^{n-k}}{(p + \beta)^n} \right],$ $G(p) = \frac{1}{2\alpha A} \left[ 2\alpha B_0 - \frac{(p - \alpha) \sum_{k=1}^n k! B_k (p + \alpha)^{n-k}}{(p + \alpha)^n} \right],$ $\alpha = \beta$

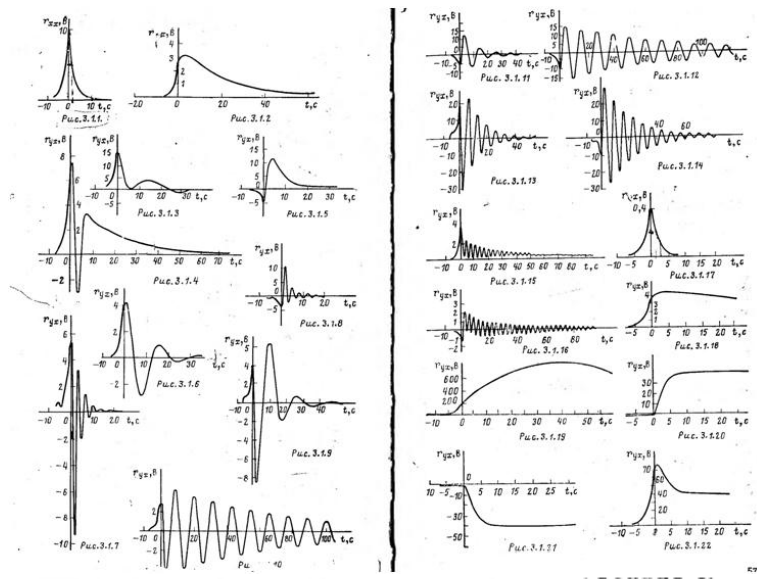
Продолжение табл. 2.1а

Дифференциальные уравнения системы	Корреляционная функция выхода
<p style="text-align: center;">5</p> $\frac{dy}{dt} + \beta y = \frac{B}{A} \frac{\alpha + \beta}{2\alpha} \left( \frac{dx}{dt} + \alpha x \right), \quad \alpha \neq \beta;$ $y = \frac{B}{A} x, \quad \alpha = \beta$	<p style="text-align: center;">6</p> $r_{yy}(t) = \frac{(\alpha + \beta)^2 B^2}{4\alpha\beta A} e^{-\beta  t }$ <p>(рис. 2.1.1)</p>
$\sum_{k=0}^{n+1} C_k \beta^{n+1-k} \frac{d^k y}{dt^k} = \frac{1}{2\alpha A} \sum_{k=0}^{n+1} F_k \frac{d^k x}{dt^k}, \quad \alpha \neq \beta;$ $\sum_{k=0}^n C_k \beta^{n-k} \frac{d^k y}{dt^k} = \frac{1}{2\alpha A} \sum_{k=0}^n F_k \frac{d^k x}{dt^k}, \quad \alpha = \beta,$ <p>где <math>F_k</math> — коэффициенты при <math>p^k</math> в числителе <math>G(p)</math></p>	$r_{yy}(t) = \frac{1}{2\alpha A} \left\{ [(\alpha + \beta) B_0 - B_1] \sum_{k=0}^n  t ^k B_k + \sum_{k=1}^{n+1} D_k \frac{1}{(2\beta)^k} \left[ B_0 (k - 1)! - \sum_{m=1}^n B_m \sum_{i=1}^{m+1} \frac{(k+m-i)!}{(2\beta)^{m-i-1}}  t ^{i-1} C_m^{i-1} \right] \right\} e^{-\beta  t },$ <p>где</p> $D_k = 2\beta k B_k + (\alpha^2 - \beta^2) B_{k-1} - k(k+1) B_{k+1}$ $r_{yy}(t) = \frac{1}{8\alpha\beta^2 A} \left\{ \frac{(\alpha^2 + \beta^2) B^2}{\beta} + (\alpha^2 - \beta^2) B_0 B_1 + [4\beta^2 (\alpha + \beta) B_0 + (2\beta B_0 + B_1) (\alpha^2 - \beta^2)] (B_0 + B_1  t ) \right\} e^{-\beta  t } \quad \text{при } n = 1$ <p>(рис. 2.1.4, 2.1.5, 2.1.6)</p>

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Номер группы	Взаимные корреляционные функции выходной и входной переменных	Дифференциальное уравнение объекта
I	Рис. 3.2.1	$r'_{yx} + 0,001r_{yx} = r_{xx}$
	Рис. 3.3.2	$r'_{yx} + 0,005r_{yx} = r_{xx}$
	Рис. 3.2.3	$r'_{yx} + 0,01r_{yx} = r_{xx}$
	Рис. 3.2.4	$r'_{yx} + 0,05r_{yx} = r_{xx}$
	Рис. 3.2.5	$r'''_{yx} + r''_{yx} + 4r'_{yx} + 0,05r_{yx} = 0, 1r''_{xx} + 0, 1r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.6	$r'''_{yx} + 3,5r''_{yx} + 4r'_{yx} + 0,1r_{yx} = 0, 1r''_{xx} + 0, 1r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.7	$r'''_{yx} + r''_{yx} + r'_{yx} + 0,05r_{yx} = 0, 1r''_{xx} + 0, 1r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.8	$r'''_{yx} + 3,5r''_{yx} + r'_{yx} + 0,05r_{yx} = 0, 1r''_{xx} + 0, 1r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.9	$r'''_{yx} + r''_{yx} + 4r'_{yx} + 0,5r_{yx} = 0, 1r''_{xx} + 0, 1r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.10	$r'_{yx} + r_{yx} = r_{xx}$
	Рис. 3.2.11	$r'''_{yx} + 3,5r''_{yx} + r'_{yx} + 0,05r_{yx} = 0, 1r''_{xx} + r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.12	$r'_{yx} + 0, 1r_{yx} = r_{xx}$
	Рис. 3.2.13	$r'_{yx} + 5r_{yx} = r_{xx}$
	Рис. 3.2.14	$r'''_{yx} + 3,5r''_{yx} + 4r'_{yx} + r_{yx} = 0, 1r''_{xx} + 0, 1r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.15	$r'_{yx} + 0, 5r_{yx} = r_{xx}$
	Рис. 3.2.16	$r'''_{yx} + r''_{yx} + 4r'_{yx} + r_{yx} = 0, 1r''_{xx} + 0, 1r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.17	$r'_{yx} + 2r_{yx} = r_{xx}$
	Рис. 3.2.18	$r'_{yx} + 10r_{yx} = r_{xx}$
II	Рис. 3.2.19	$r'''_{yx} + 3,5r''_{yx} + 4r'_{yx} + 0,05r_{yx} = 0, 1r''_{xx} + r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.20	$r'''_{yx} + 3,5r''_{yx} + r'_{yx} + 0,05r_{yx} = 5r''_{xx} + r'_{xx} + 0, 1r_{xx}$
	Рис. 3.2.21	$r'''_{yx} + r''_{yx} + r'_{yx} + 0,05r_{yx} = 0, 1r''_{xx} + r'_{xx} + 0, 1r_{xx}$

**4. Illustrative Example:**

Consider we have the following responses corresponding to the input auto-correlation graph  $r_{xx}$ ; cross-correlation graph between the input and output signals  $r_{yx}$ ; and the impulse response of the system  $g(t)$  as shown in figures 1; 2 and 3 respectively.

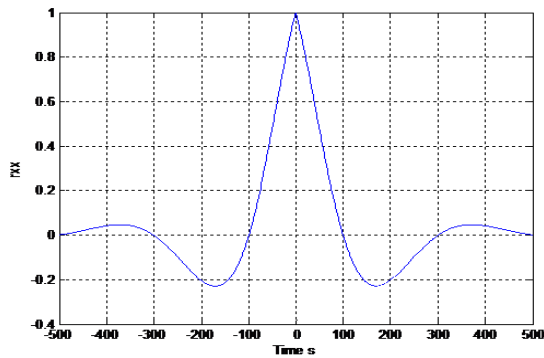


Figure 2: Autocorrelation of the input signal

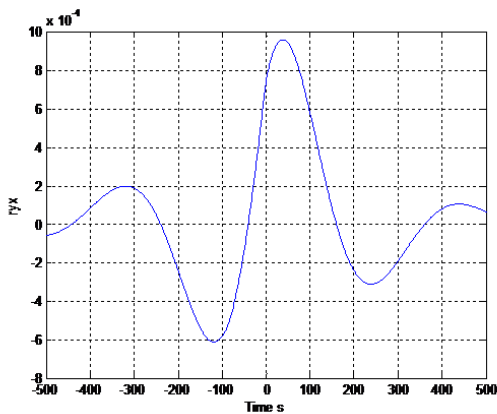


Figure 3: Cross correlation between the input and output signal

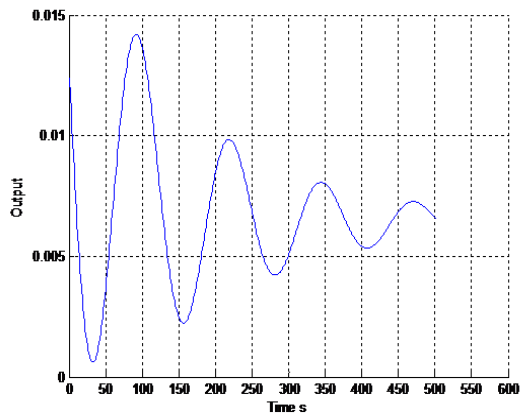


Figure 4: Impulse response of the output signal

After we check and search the given tables we have the following system equations:

$$r_{xx}(t) = e^{-0.008t} \cos\left(\frac{\pi t}{200}\right) \quad 4$$

$$r_{yx}(t) = \begin{cases} 0.76 \cdot 10^{-3} e^{-0.0056t} (\cos\frac{\pi t}{200} + 1.37 \sin\frac{\pi t}{200}), & t \geq 0; \\ 0.76 \cdot 10^{-3} e^{-0.0056t} (\cos\frac{\pi t}{200} + 1.37 \sin\frac{\pi t}{200}), & t < 0; \end{cases} \quad 5$$

$$G(S) = 8.5 \cdot 10^{-3} \frac{0.74S + 0.04}{S + 0.0176} \cdot \frac{(S + 0.008)^2 + 0.0026}{(S + 0.0056)^2 + 0.0026} \quad 6$$

$$g(\tau) = 8.5 \cdot 10^{-3} [0.76\delta(\tau) + 0.7e^{-0.0176\tau} - 2.10 \cdot 10^{-6} \cos 0.5\tau e^{-0.0056\tau} - 1.3 \sin 0.05\tau e^{-0.0056\tau}] \quad 7$$

**5. Conclusions**

In this paper we describe a very simple and practical method for structure and parameters system identification problem. This method is based on correlation information got for input and output signals and database of related correlation graphs . The main drawback of this method is low accuracy, very slow; which is completely depending on searching the graphs. And it may failed if there is no found similar signals graphs.

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