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ON STRONG B(m,n) NEAR SUBTRACTION SEMIGROUPS

V.MAHALAKSHMI[1], S.MAHARASI[2], S.JAYALAKSHMI[3],

[1]Research Scholar, Sri Parasakthi College for Women, Courtallam, Tamil Nadu,India

[2]Assistant Professor in Mathematics,V.V College Of Engineering, Tisayanvilai, Tamil Nadu,India

[3]Associate professor in Mathematics,Sri Parasakthi College for Women, Courtallam Tamil Nadu,India

ABSTRACT

In this paper, we introduced the concept of B (m,n) near subtraction semigroup. (i.e) A near subtraction semi group X has the property B(m,n), if there exist positive integers m, n such that $<x>_r{}^m X = X<x>_l{}^n$, for all x in X. We also discuss some of their properties and obtained certain theorem.

Key words: $\bar{s}$-near subtraction semigroup, property (α), idempotent, regular.

1. INTRODUCTION

B. M. Schein [10] considered systems of the form (X; **0; **), where X is a set of functions closed under the composition "**0**" of functions (and hence (X; **0**) is a function semigroup) and the set theoretic subtraction "****" (and hence (X; ****) is a subtraction algebra in the sense of [1]).He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B.Zelinka [11] discussed a problem proposed by B. M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y.B.Jun [5] introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. In [4], Y. B. Jun and H.S.Kim established the ideal generated by a set, and discussed related results. For basic definition one may refer to Pilz[8].

In this paper, with a new idea, we define strong B(m,n) near subtraction semigroup and investigate some of their properties. After deriving basic properties of a strong B(1,2) near subtraction semigroup, We obtain necessary and sufficient condition for a s-near subtraction semigroup with property (α) to be a strong B(1,2) near subtraction semigroup. It is also shown that, in a strong B(m,n) s-near subtraction semigroup with property (α), the concepts of prime ideal, completely prime ideal, and maximal ideal coincide. Unless stated otherwise throughout this paper X stands for a zero-symmetric near subtraction semigroup.

2. Preliminaries on Near Subtraction Algebra

Definition 2.1: A non-empty set X together with binary operations "–" and is said to be a **subtraction algebra** if it satisfies the following:

I. $x-(y-x)=x$.
II. $x-(x-y)=y-(y-x)$.
III. $(x-y)-z=(x-z)-y$. for every $x,y,z\in X$.

Definition 2.2: Anon-empty set X together with two binary operations "–" and "•" is said to be a **subtraction semigroup** if it satisfies the following:

I. (X,–) is a subtraction algebra.
II. (X,•)is a semigroup.
III. $x(y-z) = xy-xz$ and $(x-y)z = xz-yz$,for every $x,y,z\in X$.

Definition 2.3: A non-empty set X together with two binary operations "–" and "•" is said to be a **near subtraction semigroup (right)** if it satisfies the following:

I. (X,–) is a subtraction algebra.
II. (X,•) is a semigroup.
III. $(x-y)z = xz-yz$, for every $x,y,z \in X$.

Definition 2.4: A non-empty subset S of a subtraction semigroup X is said to be a **sub algebra** of X, if $x-x' \in S$ when ever $x,x' \in S$.

Note 2.5: Let X be a near subtraction semigroup. Given two subsets A and B of X, $AB=\{ab/a \in A, b \in B\}$.Also we defined another operation " *",
$A*B = \{ab-a(a'-b)/a,a' \in A, b \in B\}$.

Definition2.6: An element $e \in X$ is said to be **idempotent** if for each $e \in X$, $e^2 = e$.

Definition 2.7: We say that X is an **s(s') near subtraction semigroup** if $a \in Xa(aX)$, for all $a \in X$.

Definition 2.8: A s-near subtraction semigroup X is said to be a **$\bar{s}$-near subtraction semigroup** if $x \in xX$, for all $x \in X$.

Definition 2.9: A near subtraction semigroup X is said to be **sub commutative** if $aX= Xa$, for every $a \in X$.

Definition 2.10: A near subtraction semigroup X is said to be **left-bipotent** if $Xa=Xa^2$, for every $a \in X$.

Definition2.11: An element $a \in X$ is said to be **regular** if for each $a \in X$, $a=ab\,a$ for some $b \in X$.

Definition2.11: A near subtraction semigroup X is called **strongly regular** if for each $a \in X$, there exists $b \in X$ such that $a = ba^2$.

Note 2.13: Let X be a zero-symmetric near subtraction semigroup and if X is strongly regular, then X is regular.

Definition 2.14:A near subtraction semigroup X is said to have **property(α)** if xX is a subalgebra of (X,–), for every $x \in X$.

Definition 2.15: A subalgebra A of (X,–) is called an **left(right)X-subalgebra** of X if $A(AX) \subseteq A$.

Note 2.16: Let $a \in X$. Then $<a>_r(<a>_l)$ is the intersection of all right(left) X-subalgebra containing a.

Definition 2.17:A nearsubtraction semigroupX is said to be **two sided** if every left X-subalgebra is right X-subalgebra and vice versa.

Note 2.18: Whenever a zero–symmetric nearsubtraction semigroup contains non-zero nilpotent elements, then X has IFP.

Definition2.19: Let P be an ideal of X. P is called

(i) a **prime ideal,** if for all ideals I, J of X, $IJ \subseteq P \Rightarrow I \subseteq P$ or $J \subseteq P$.
(ii) a**completely prime ideal,** if for any a, b in X, $ab \in P \Rightarrow$ either $a \in P$ or $b \in P$.
(iii) a **primary ideal** if $abc \in P$ and if the product of any two of a, b, c is not in P, then the k^{th} power of the third element is in P.
(iv) a **maximal ideal** (**minimal ideal**) if it is **maximal** (**minimal**) in the set of all non-zero ideals of X.

3. On Strong B(m,n) near subtraction semigroups

In this section, We discuss some properties of strong B(1,2) near subtraction semigroup and some properties of strong B(1,2) $\bar{s}$-near subtraction semigroup with property (α).

Definition3.1: We say that a near subtraction semigroup X has the property Strong B(m,n), if there exist positive integers m, n such that $<x>_r^m a = a<x>_l^n$, for all x,a in X.

Example3.2.1: Let X=**{**0,a,b,1**}**in which**"–"**and**"•"**are defined by,

–	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

•	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Hence X is a strong B(m,n) near subtraction semigroup, for all positive integers m and n.

Example3.2.2: Let X={0,a,b,c} in which **"–"** and **"•"** a redefined by,

–	0	a	b	c
0	0	0	0	0
a	a	0	a	b
b	b	b	0	b
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	0	b
c	0	a	0	c

X is a B(2,1)near subtraction semigroup. But not a strong B(2,1)near subtraction semigroup, Since $<c>_r{}^2 b \neq b<c>_l$.

Example:3.2.3 Let X={0,a,b,c} in which "–" and "•"are defined by,

–	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	b
c	c	0	c	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	c	b
c	0	a	b	c

X is a strong B(2,3)near subtraction semigroup. But not a strong K(2,3)near subtraction semigroup, Since $b^2c \neq cb^3$.

Proposition 3.3: Every left X-subalgebra of strong B(1,2)near subtraction semigroup is also a rightX-subalgebra.

Proof: Let A be a left X-subalgebra of X and $a \in A$. Since X is a strong B(1,2)near subtraction semigroup, for $x \in X$, $ax \in <a>_r x = x<a>_l{}^2 \subseteq x<a>_l \subseteq <a>_l \subseteq A$. (i.e.,) $AX \subseteq A$. Hence A is a right X-subalgebra of X.

Proposition3.4: If X is a strong B(2,1) near subtraction semigroup, then every right X-subalgebra is also a left X-subalgebra.

Proof: Let A be a right X-subalgebra of X. Then we have $AX \subseteq A$. Since X is a strong B(2,1)near subtraction semigroup, for $a \in A$ and $x \in X$, $xa \in x<a>_l = <a>_r{}^2 x \subseteq <a>_r x \subseteq <a>_r \subseteq A$. (i.e.,) $XA \subseteq A$. Hence A is a left X-subalgebra of X.

Proposition 3.5: If X is a strong B(1,2) and strong B(2,1)near subtraction semigroup, then X is two sided.

Proof: Follows from Propositions 3.3 and 3.4.

Proposition3.6: Let X be a $\bar{s}$-near subtraction semigroup with property (α). If X is a strong B(1,2) near subtraction semigroup, then X has strong IFP.

Proof: Let I be an ideal of X. Assume that $ab \in I$ for a, $b \in X$. For $a,x \in X$, $ax \in <a>_r x = x<a>_l{}^2 \in X<a>_l{}^2 \subseteq <a>_l{}^2 = XaXa$ and so $ax = x_1ax_2a$. Thus $axb = x_1ax_2ab \in XI \subseteq I$ and so $axb \in I$. (i.e.,) X has strong IFP.

Proposition 3.7: Let X be a $\bar{s}$-near subtraction semigroup with property (α). If X is a strong B(1,2) near subtraction semigroup, then $M_1 \cap M_2 = M_1M_2$, for any two left X-subalgebra M_1 and M_2 of X.

Proof: Let $x \in M_1 \cap M_2$. Then, $x^2 \in <x>_r x = x<x>_l{}^2 = xXxXx \in XXxXx \subseteq XxXx \subseteq XM_1XM_2 \subseteq M_1M_2$. (i.e.,)$M_1 \cap M_2 \subseteq M_1M_2$.On the other hand, if $x \in M_1M_2$ then $x = yz$, Where $y \in M_1$ and $z \in M_2$.

Now, we have $x = yz \in <y>_r z = z<y>_l{}^2 \in X<y>_l{}^2 \subseteq <y>_l{}^2 = XyXy \subseteq Xy \in XM_1 \subseteq M_1$. (i.e.,)$M_1M_2 \subseteq M_1$.Similarly $M_1M_2 \subseteq M_2$ and so $M_1M_2 \subseteq M_1 \cap M_2$.

Proposition3.8: Let X be a strong B(1,2) $\bar{s}$-near subtraction semigroup with property (α). Then $Xx \cap Xy = Xxy$, for all x,y in X.

Proof: Let x, $y \in X$. Taking $M_1 = Xx$ and $M_2 = Xy$ in the above Proposition 3.7, we get $Xx \cap Xy = XxXy$. Also $Xx = Xx \cap X = XxX$ and this yields that $Xxy = XxXy$. Hence $Xx \cap Xy = Xxy$.

Proposition 3.9: Let X be a strongB(1,2) $\bar{s}$-near subtraction semigroup with property (α). Then X is left bi-potent.

Proof: By the Proposition 3.8, for $a \in X$, We have $Xa = Xa \cap Xa = Xaa = Xa^2$.

(i.e.,) X is left bi-potent.

Corollary3.10: Let X be a strong B(1,2) $\bar{s}$-near subtraction semigroup with property (α). Then X is strongly regular.

Proof: Trivially follows from the fact that $a \in Xa$ and from Proposition 3.9.

Corollary3.11: Let X be a strongB(1,2) $\bar{s}$-near subtraction semigroup with property (α). Then X is regular.

Proof: Follows from theCorollary 3.10&Note 2.13.

Theorem3.12: Let X be a strong B(1,2) $\bar{s}$-near subtraction semigroup with property (α) and let A and B be any two left X-subalgebras of X. Then we have the following.

I. $\sqrt{A} = A$.

i. $A \cap B = AB$.

ii. $A^2 = A$.

II. If $A \subset B$, then $AB = A$.

III. $A \cap XB = AB$.

IV. If A is proper, then each element of A is a zero divisor.

V. A is a completely semi prime ideal of X.

Proof:

I. For $x \in \sqrt{A}$, there exists some positive integer k such that $x^k \in A$. Since X is a strong B(1,2)$\bar{s}$-near subtraction semigroup with property (α), by the Corollary 3.10, X is strongly regular. If $x \in X$, then $x = ax^2$, for some $a \in X$. This implies $x = ax^2 = (ax)x = a(ax^2)x = a^2 x^3 = \ldots\ldots\ldots = a^{k-1} x^k \in XA \subseteq A$. (i.e.,) $x \in A$. Thus $\sqrt{A} \subseteq A$. Obviously $A \subset \sqrt{A}$ and so $A = \sqrt{A}$.

II. Since X is a $\bar{s}$-near subtraction semigroup with property (α), by the Proposition 3.7, $AB = A \cap B$.

III. Taking B = A in (ii), We get $A = A^2$. Suppose that $A \subset B$. Then $A \cap B = A$ and(ii) gives A = AB.

IV. $A \cap XB \subset A \cap B$ and so $A \cap XB \subset AB$ (by (ii)). Also $AB = A \cap B \subset A$ and $AB \subset XB$. Therefore $AB \subset A \cap XB$. Hence $AB = A \cap XB$.

V. If X has the IFP, the concepts of left zero-divisors, right zero-divisors and zero-divisors are equivalent in X. Thus we need only to prove that A* consists of only zero-divisors. Let $a \in A$. By (iii), for the principal left X-subalgebraXa, $Xa = (Xa)^2 = XaXa$. Consequently, for any $x \in X$, there exists $y,z \in X$such that xa = yaza. (i.e.,) (x–yaz)a = 0. If a is not a zero-divisor, then x–yaz = 0. This implies $x = yaz \in XAX \subset A$. (i.e.,) $X \subset A$. Hence X = A which is a contradiction to the hypothesis that A is proper. Thus $a \in A^*$. Hence 'a' is a zero-divisor.

VI. Let $a^2 \in A$. By the Proposition 3.6, X has strong IFP. So $axa \in A$.By Corollary 3.11,$a \in A$. Hence A is completely semi-prime.

Theorem3.14: Let X be a strong B(1,2) $\bar{s}$-near subtraction semigroup with property (α) and let P be a proper left X-subalgebra of X . Then the following are equivalent.

I. P is a prime ideal.
II. P is a completely prime ideal.
III. P is a primary ideal.
IV. P is a maximal ideal.

Proof:

(i) ⇒ (ii)

By Remark 2.2.23,P is an ideal of X and assume that P is a prime ideal. Let $ab \in P$. By the Proposition 3.8 and 2.7.22, $XaXb = Xab \in XP \subseteq P$. By the Remark 2.2.23, Xa and Xb are ideals in X. Since P is prime, $XaXb \subseteq P$ which implies $Xa \subseteq P$ or $Xb \subseteq P$. Suppose $Xa \subseteq P$, then $a = axa \in Xa \subseteq P$. Similarly $Xb \subseteq P$ gives that $b = byb \in Xb \subseteq P$ and (ii) follows.

(ii) ⇒ (i) Obvious.

(ii) ⇒ (iii)

By the Proposition 3.8, for all $x, y \in X$, $Xxy = Xx \cap Xy$. As $Xx \cap Xy = Xy \cap Xx = Xyx$, We see that Xxy = Xyx, for all $x, y \in X$. Using this we get that, for all $a, b, c \in X$, Xabc = Xbca = Xcab = Xacb = Xbac = Xcba. Suppose that $abc \in P$ and $ab \notin P$. Since Xis a strong B(1,2) $\bar{s}$-near subtraction semigroup with property (α),by the Corollary 3.11, X is regular.Therefore $abc = axabc \in Xabc \subseteq XP \subseteq P$ and therefore $(ab)c \in P \Rightarrow c \in P$. (as P is a completely prime ideal and since $ab \notin P$). Again suppose $abc \in P$ and $ac \notin P$. To get the desired result we proceed as follows. Now $acb \in Xacb = Xabc \subseteq XP \subseteq P$. Thus $acb = (ac)b \in P$ and If $ac \notin P$, then $b \in P$ as before. Continuing in this way it is easy to prove that if $abc \in P$ and if the product of any two of a,b,c does not fall in P, then the third falls in P. This proves (iii).

(iii) ⇒ (ii)

Let $ab \in P$ and $a \notin P$. First we observe that $xa \notin P$, for $x \in X$ satisfying a = axa. For, $xa \in P \Rightarrow a = a(xa) \in XP \subseteq P$ which is a contradiction. Also $xab \in XP \subseteq P$. Thus $xab \in P$ and $xa \notin P$. Since P is a primary ideal of X, $b^k \in P$, for some positive integer k. Now $b^k \in P \Rightarrow b \in \sqrt{P}$andby Theorem 3.12$\sqrt{P}$= P. Thus $b \in P$ and (ii) follows.

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