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RESEARCH ARTICLE

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ENERGY-BASED AVERAGING BY APPLYING DIRECTIVE VDI 4675 TO VARIOUS FLOW SAMPLE CASES

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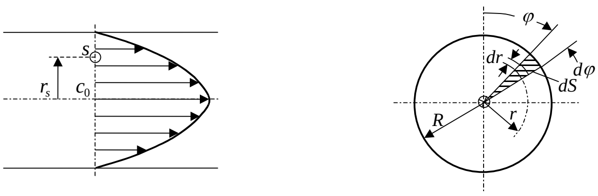
ABSTRACT

In many cases numerical data reduction involving integration relies on numerical methods leaving the design engineer with the uncertainty if the spread in the conservation of a flux is caused by the numerical integration and differentiation in the data reduction or if the method of averaging exhibits deficiencies. For this reason, the averaging methods are suitably tested in problems with analytically obtained results to circumvent the question of influence of approximative solutions. In this contribution laminar and turbulent pipe flow is treated with an averaging method put down in directive VDI 4675 for balance-based averaging assuming that power laws describe the time-independent distribution of incompressible flow. Laminar flow relies on a parabolic distribution across the through flow section and turbulent flow is considered to be represented by the 1/7-power law of boundary layers. The final result of these simple cases are accessible to analytical integration and show that the situation can be handled to fulfill the system of balance equations. Nevertheless, not all solutions show to be representative for physical behaviour of the diffusion though energy conservation is achieved.

Keywords—average field values, field average, energy balance

Laminar flow-field distribution



Rotationally symmetric laminar flow without swirl

Definition of the situation [1]

Pressure and specific enthalpy

$$p = const.;\ h = const.$$

Density and specific entropy

$$\rho, s = f(p,h) \neq f(r,\varphi)$$

velocity $c_n = c = c_0 \left[1 - \left(\frac{r}{R} \right)^2 \right]$ $\qquad c_r = 0 \quad , \quad c_\varphi = 0$

Elemental cross section $dS = d\varphi \cdot \frac{R^2}{2}$

Conservative fluxes in an elemental section $d\varphi$ (balance of flux is retained)

Mass flux $d\dot{m} = \int_0^R \rho c r \, dr \, d\varphi = d\varphi \cdot \rho c_0 \frac{R^2}{4}$

Flux of total linear momentum $d\dot{I}_{0_n} = \int_0^R p r \, dr \, d\varphi + \int_0^R \rho c^2 r \, dr \, d\varphi = d\varphi \cdot p \frac{R^2}{2} + d\varphi \rho c_0{}^2 \frac{R^2}{6}$

Flux of total enthalpy $d\dot{H}_0 = \int_0^R \left(h + c^2/2 \right) \rho c r \, dr \, d\varphi = d\dot{H} + d\dot{K}$

Non-conservative fluxes in an elemental section $d\varphi$ (integral values of the fluxes are maintained, some variables are changed to irreversibly averaged " ^ "-values which fulfill the balance equations)

Flux - of static enthalpy - of kinetic energy - of entropy $\dot{S}$ in the elemental section dS

$$d\dot{H} = d\varphi \cdot h \cdot \rho c_0 \frac{R^2}{4} \quad ; \quad d\dot{K} = d\varphi \rho c_0{}^3 \frac{R^2}{16} \quad ; \quad d\dot{S} = \int_0^R s \rho c r \, dr \, d\varphi = d\varphi \, s \rho c_0 \frac{R^2}{4}$$

Irreversibly averaged quantities (^) originate in a mixing process which increases entropy in the conversion to equalization.

$$d\dot{m} = d\varphi \cdot \rho c_0 \frac{R^2}{4} = \rho \hat{c}_n dS \quad ; \quad \hat{c}_n = \frac{c_0}{2} \quad ;$$

$$d\dot{I}_{0_n} = d\varphi p \frac{R^2}{2} + d\varphi \rho c_0{}^2 \frac{R^2}{6} = \hat{p} \, dS + \rho \hat{c}_n{}^2 dS \quad ; \quad \hat{p} = \frac{\rho c_0{}^2}{12} + p$$

If the pressure $\hat{p}$ is defined from the preceding equation for the total momentum flux through an elemental section, averaging leads to an increase by $\rho c_0^2/12$ for the static pressure and an equivalent decrease in the dynamic pressure. In this way total momentum is maintained constant.

$$d\dot{H}_0 = d\varphi \cdot h \cdot \rho c_0 \frac{R^2}{4} + d\varphi \cdot \rho c_0{}^3 \frac{R^2}{16} = \hat{h} \rho \hat{c} dS + \frac{\hat{c}^2}{2} \cdot \rho \hat{c} dS \; ; \; \hat{h} = \frac{c_0{}^2}{8} + h \; ; \; \hat{h}_0 = \hat{h} + \frac{\hat{c}^2}{2}$$

In the definition of the mean value of the specific enthalpy $\hat{h}$ from the equation for the flux of total enthalpy the static part increases by $c_0^2/8$ and decreases the dynamic part by the same amount. The requirement of constant flux of total enthalpy is thus fulfilled.

Reversibly averaged quantities (~) originate isentropically from the integration of the corresponding inhomogeneous flux equation by integration of the corresponding balance equation,

$$\dot{H} = \frac{1}{2}\cdot\rho\cdot c_0\cdot h\cdot S = \frac{1}{2}\cdot\rho\cdot c_0\cdot\tilde{h}\cdot S \quad [W] \qquad \text{leads to} \qquad \underline{\tilde{h} = h} \quad \left[\frac{Nm}{kg} = \frac{m^2}{s^2}\right],$$

$$\dot{K} = \frac{1}{8}\cdot\rho\cdot c_0^{\,3}\cdot S = \frac{1}{2}\cdot\rho\cdot c_0\cdot\tilde{k}\cdot S \quad [W] \qquad \text{leads to} \qquad \underline{\tilde{k} = \frac{1}{4}\cdot c_0^{\,2}} \quad \left[\frac{Nm}{kg} = \frac{m^2}{s^2}\right],$$

$$\dot{S} = \frac{1}{2}\cdot\rho\cdot c_0\cdot s\cdot S = \frac{1}{2}\cdot\rho\cdot c_0\cdot\tilde{s}\cdot S \quad [W] \qquad \text{leads to} \qquad \underline{\tilde{s} = s} \quad \left[\frac{Nm}{kg} = \frac{m^2}{s^2}\right].$$

Characteristic numbers

- for the irreversibly averaged flow the Reynolds number is $\hat{Re} = \dfrac{2R\cdot\hat{c}_n\cdot\rho}{\hat{\eta}(\hat{p},\hat{h})}$
- for the reversibly averaged mean values three characteristic numbers indicate the distribution of the reversibly averaged kinetic energy into contributions to the other balance equations:

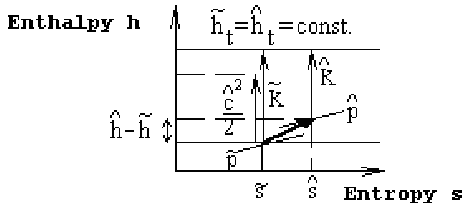
$$\xi = \frac{\hat{k}}{\tilde{k}} = \frac{1}{2} \quad ; \qquad \chi = \frac{\hat{p}-\tilde{p}}{\rho\,\tilde{k}} = \frac{1}{3} \quad ; \qquad \zeta = 1-\xi-\chi = \frac{1}{6}$$

In this special case they are:

residual kinetic energy $\hat{k}$ pressure recovery $\hat{p}-\tilde{p}$ and apparent dissipation j

$$\xi = \frac{\hat{k}}{\tilde{k}} = \frac{\frac{1}{8}C^2}{\frac{1}{4}C^2} = \frac{1}{2} \quad ; \qquad \chi = \frac{\hat{p}-\tilde{p}}{\rho\tilde{k}} = \frac{\frac{1}{12}\rho C^2}{\rho\frac{1}{4}C^2} = \frac{1}{3} \quad ; \qquad \zeta = \frac{j}{\tilde{k}} = 1-\chi-\xi = \frac{1}{6}$$

The averaged situation in the h-s-diagram is now described by two sets of variables (~ , ^) of which both have the same specific total enthalpy:



(^) describes the one-dimensional situation with increase in entropy due to diffusion by mixing,
(~) describes the one-dimensional situation after integration, which can originate only isentropically, i.e. without increase in entropy due to mixing or equalizing

With the use of the Reynolds number $\hat{Re} = \dfrac{2R\cdot\hat{c}_0\cdot\rho}{\hat{\eta}(\hat{p},\hat{h})}$ and assuming the apparent specific dissipation $j = \zeta\tilde{k}$ for equalizing the inhomogeneous situation to be identical to the viscous dissipation $j_\tau = \dfrac{\tau}{\rho}$ in the wall shear stress of the boundary layer we obtain with the molecular viscosity η

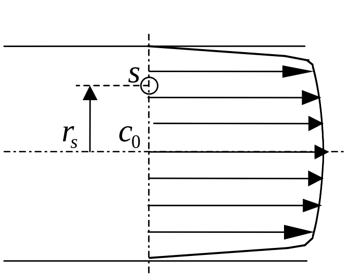
$$\zeta = \frac{j_\tau}{\tilde{k}} = \frac{\frac{1}{2\pi R}\int_0^{2\pi}\int_0^R \left|\frac{\tau}{\rho}\right| dr\, d\varphi}{\tilde{k}} = \frac{\eta}{\rho \tilde{k}}\frac{dc_n}{dr} = \frac{\eta}{R^2\rho}\frac{2C_0}{\tilde{k}}\frac{1}{2\pi R}\int_0^{2\pi}\int_0^R r\,dr\,d\varphi = \frac{\eta}{\rho\tilde{k}}\frac{C_0}{R} = \frac{\eta C_0^2}{\rho\tilde{k}C_0 R} =$$

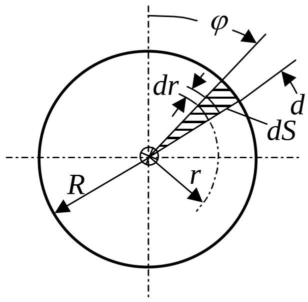
$$= \frac{4\eta C_0^2}{\rho R C_0^2 C_0} = \frac{4\eta}{\frac{D}{2}\rho 2\hat{c}} = \frac{4}{\hat{Re}} = \frac{1}{6}$$ and the identity shows to be valid for the Reynolds number

$\hat{Re} = 24$ in one-dimensional pipe flow with $\hat{h}_t = h_t$.

The parabolic velocity distribution of the incompressible laminar pipe flow and its one-dimensional mean values (^) are compatible with a physical situation only for the Reynolds number $\hat{Re} = 24$.

Turbulent flow-field distribution





Rotationally symmetric turbulent flow without swirl

The condition for turbulent incompressible pipe flow is described by a velocity distribution according to the 1/7-power law

Pressure and specific enthalpy

$p = \text{const.};\ h = \text{const.}$

Density and specific entropy

$\rho, s = f(p,h) \neq f(r,\varphi)$

Velocity $c_n = c = c_0\left[1 - \frac{r}{R}\right]^{\frac{1}{7}}$

$c_r = 0 \quad ; \quad c_\varphi = 0$

Elemental cross section $dS = d\varphi \cdot \frac{R^2}{2}$

These conditions are introduced into the respective flux equation and treated in analogy to the aforementioned laminar case:

Irreversibly averaged mean values generated by the integration of the respective flux_$\dot{\Phi}$ from the balance equation $\int d\dot{\Phi} \equiv \dot{\hat{\Phi}}$ solved for $\hat{\Phi} - \Phi = \cdots$ lead to

$\hat{c}_n = \frac{49}{60} c_0$ from $\int d\dot{m} \equiv \dot{m} = \rho \hat{c} S$ by integration with the substitution $z = \left[1 - \frac{r}{R}\right]$;

$\frac{\hat{p} - p}{\rho} = c_0^{\,2} \left[\frac{49}{72} - \left(\frac{49}{60}\right)^2\right]$ from $\int d\dot{I}_0 \equiv \hat{\dot{I}}_0$;

$\hat{h} - \tilde{h} = \frac{c_0}{2}\left[\frac{12}{17} - \left(\frac{49}{60}\right)^2\right]$ from $\int d\dot{H}_0 \equiv \dot{m}\hat{h} + \dot{m}\hat{k}$;

$\tilde{k} = \frac{12}{17} \frac{c_0^{\,2}}{2}$ $\int d\dot{K} \equiv \dot{m}\tilde{k}$; $\hat{k} \equiv \frac{\hat{c}}{2} = \left(\frac{49}{60}\right)^2 \frac{c_0^{\,2}}{2}$;

The redistribution of the reversibly obtained kinetic energy $\tilde{k}$ is shown by

$$1 = \xi + \chi + \zeta = \frac{\hat{k}}{\tilde{k}} + \frac{\hat{p} - p}{\rho \tilde{k}} + \frac{j}{\tilde{k}}$$

With the 1/7-power law for the turbulent velocity distribution the above-mentioned characteristic numbers define a redistribution of the reversibly averaged kinetic energy into:

-residual kinetic energy $\hat{k}$: -pressure recovery $\hat{p} - \tilde{p}$:

$$\xi = \frac{\hat{k}}{\tilde{k}} = \frac{\left(\frac{49}{60}\right)^2 \frac{C_0^{\,2}}{2}}{\left(\frac{12}{17}\right)\frac{C_0^{\,2}}{2}} = \left(\frac{49}{60}\right)^2 \frac{17}{12} = 0{,}9448\ ; \quad \chi = \frac{\hat{p} - \tilde{p}}{\rho \tilde{k}} = \frac{C_0^{\,2}\left[\frac{49}{72} - \left(\frac{49}{60}\right)^2\right]}{\frac{C_0^{\,2}}{2}} \frac{17}{12} = 0{,}0386;$$

and apparent dissipation j :

$$\zeta = \frac{j}{\tilde{k}} = 1 - \xi - \chi = 1 - 0{,}9448 - 0{,}0386 = 0{,}0166$$

The latter is the apparent loss coefficient due to mixing in the averaged one-dimensional balance.

Employing wall shear stress relations of the incompressible developed pipe flow a situation can be specified in which the apparent loss coefficient of the averaging method is compatible with the physical loss coefficient

$$\zeta = \frac{j_\tau}{\tilde{k}} = \frac{|\tau|}{\rho \tilde{k}} = \frac{\eta\, dc}{\rho\, k\, dr} = \frac{\eta\, C_0}{7\rho\, \tilde{k} R} = \frac{\eta\, C_0}{7\rho \frac{C_0^{\,2}}{2} R} \frac{17}{12} = \frac{1}{Re} \frac{17}{3} = \frac{1}{\hat{Re}} \frac{17}{3} \frac{49}{60} \equiv 0{,}0166$$

Assuming that ζ of the averaging method is identical with the physical friction loss in the wall layer, the corresponding Reynolds number adopts the value

$$\hat{Re} = \frac{4{,}628}{0{,}0166} \approx 279\ .$$

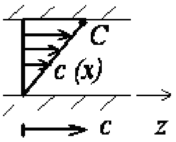
The 1/7-power law for the velocity distribution in incompressible turbulent pipe flow and its irreversibly averaged one-dimensional values (^) are physically compatible with wall shear flow only for the Reynodls number $\hat{Re} \approx 279$. At other values of the Reynolds number the set of irreversibly averaged quantities do conserve the equation of total energy but the model does not represent a physical situation with respect to loss.

Characteristic numbers for various cases and their respective diffusion (values are truncated)**:**

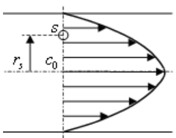
- irreversibly averaged residual kinetic energy $\hat{k}$
- pressure recovery $\dfrac{\hat{p}-\tilde{p}}{\rho}$
- dissipation j

- rated with the reversibly averaged kinetic energy $\tilde{k}$ of the inhomogeneous flow:

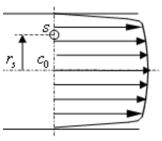
$$\xi = \frac{\hat{k}}{\tilde{k}} \qquad \chi = \frac{\hat{p}-\tilde{p}}{\rho \tilde{k}} \qquad \zeta = \frac{j_\tau}{\tilde{k}}$$



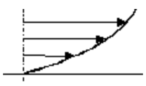
- linearly distributed laminar shear flow [1]: 0,50 0,33 0,17



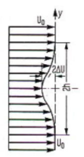
- laminar incompressible pipe flow without swirl (parabolic velocity distribution), [1] 0,50 0,33 0,17



- incompressible turbulent pipe flow without swirl (1/7-power law velocity distribution, [4]): 0,94 0,04 0,02
($\hat{R}e_D = 279$)



- 2D laminar parabolic boundary-layer ($\hat{R}e_D = 7$) [5] : 0,259 0,415 0,326



- cosine-shaped wake: 0,034 0,011 0,954
([6], $\hat{R}e_D = 2{,}1$)

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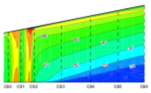
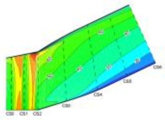
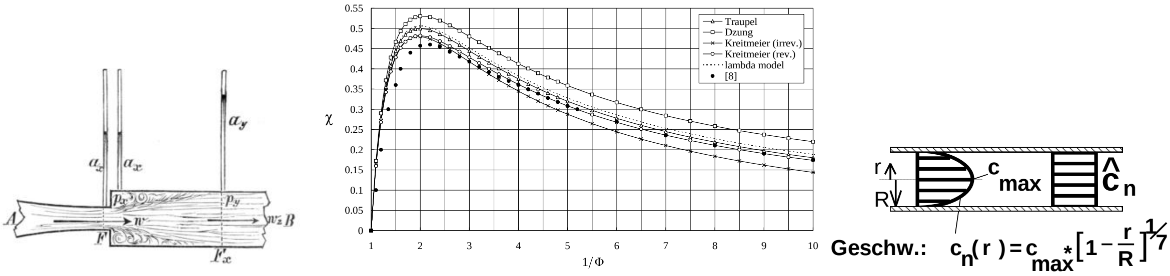
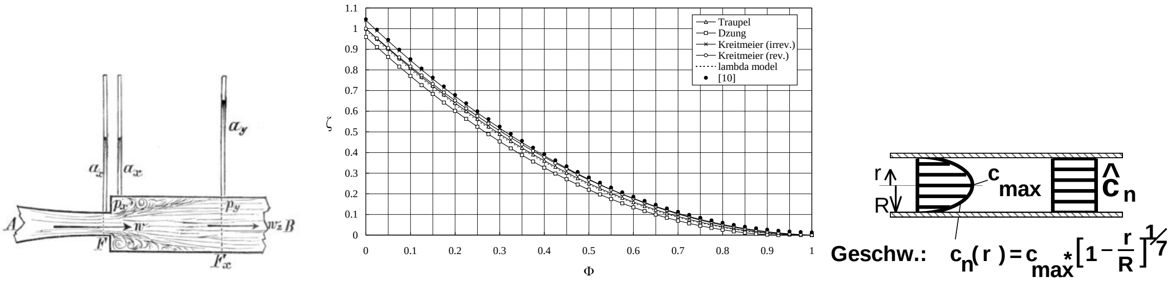
		ξ	χ	ζ
	- standard diffuser [7]:	0,48	0,32	0,20
	- optimised diffuser [7]:	0,71	0,17	0,12

The pressure ratio distribution in the sudden expansion of a Carnot-shock diffuser [figure. 6, page 21, from "Vorlesungen über Theorie der Turbinen", Prof. Dr. Gustav Zeuner (retired), publisher: A.Felix, Leipzig 1899]



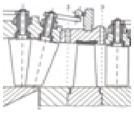
The pressure recovery coefficient χ in the Carnot shock diffuser is shown over the area ratio **1/Φ** from outlet to inlet section. The data is obtained by employing several averaging methods in comparison with the experiment [8] using the 1/7-power law velocity distribution. The left corner corresponds to pipe flow with constant diameter, the right corner is a sudden enlargement of 10 times the cross section. The symbols indicate the averaging methods: Δ Traupel; □ Dzung; **X** Kreitmeier (reversibel); ○ Kreitmeier (irreversibel); ● General Electric 1962 [8]; ----- lambda-model [9] .



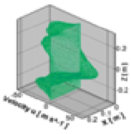
The pressure loss coefficient ζ in the Carnot shock diffuser displayed over the ratio **Φ** of inlet to outlet cross section evaluated with several averaging methods in comparison with the measurement [10] using the 1/7-power law velocity distribution. The left corner corresponds to a sudden enlargement of infinite cross section, the right corner corresponds to a pipe flow of constant diameter.

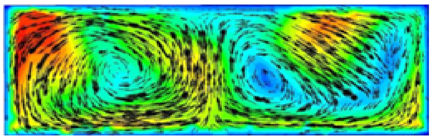
Symbols signify the respective averaging method: Δ Traupel; □ Dzung; X Kreitmeier (reversible); ○ Kreitmeier (irreversible); ----- lambda model [9]; ● Idelchik [10]

ξ χ ζ

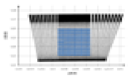


- hot-film measurements in the wake of rotor 1 of an axial compressor [11]: 0,95 0,03 0,02





- in homogenous velocity distribution in a steam turbine outlet model [4] viewed in a plane cross section (right) and as a 3D velocity profile (left), ITSM, Univ. of Stuttgart [12], [4]: 0,41 0,16 0,43 .



-steam turbine through flow computation outside the wall boundary layer [13]: 0,63 0,28 0,09

CONCLUSION

The result of the averaging method, which implies conservation of total energy, momentum and mass specifies how far the irreversibly averaged one-dimensional quantities (^) deviate from their corresponding individual integral balance by the value of its characteristic numbers. The dissipative contribution in the irreversibly averaged state shows entropy increase due to equalizing of inhomogeneous distributions which are physically real only in special situations where the viscous dissipation is accidentally equivalent to artificial diffusion of mixing.

The Reynolds number $\hat{\mathrm{Re}}_D$ for which the loss coefficient of the molecular diffusion $\zeta_{molecular}$ equals the artificial diffusion $\zeta_{averaging}$ of the equalization for averaging the mean values (^) correspond to a physically valid solution for the inhomogeneous flow field. Else the average values are mathematically valid without physical correctness. This can be easily shown here for the case of constant diameter pipe flow.

Whether the experimental values are to be attributed to irreversibly averaged data or to reversibly averaged values depends on the amount of diffusion acting in the location of measurement. Pressure probe data from strongly inhomoheneous flow regions correspond more closely to irreversibly averaged values, data from homogeneously distributed flow correspond more closely to reversibly averaged mean values in the method of F.Kreitmeier. This feature is seen in results for irreversibly averaged values from Carnot diffuser data for the range of area ratios 2,5<exit / inlet<3 as well as for reversibly averaged data for the range of area ratio 4<exit/inlet.

The result depends also on the applicability of the 1/7-power law velocity distribution for the case of the Carnot diffuser.

In the more evenly distributed turbulent pipe flow the required amount of redistribution of energy is less important than in the case of the laminar pipe flow distribution and the case of extremely inhomogeneous flow in the turbine discharge duct.

Acknowledgement

F. Kreitmeier (Alstom Power) is acknowledged for help in the use of the computer program HMITTL, C. Musch (Siemens Energy) and D. Engelmann for assistance in the preparation of 3D flow-field computer results as well as H.-G. Hencke (VDI) with the working group VDI 4675 for permission to use the computer code for student project work.

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