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RESEARCH ARTICLE

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RECTANGLE WITH AREA AS A SPECIAL POLYGONAL NUMBER

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ABSTRACT

Patterns of Rectangles in each of which the area is represented by some polygonal number. A few relations among the solutions and special polygonal numbers namely, Triangular, Hexagonal, Octagonal, Decagonal, Hexadecagonal, Octadecagonal numbers and Gnomonic, Star, Centered Hexagonal and Truncated Tetrahedral numbers are presented.

Keywords: Rectangle, Polygonal numbers, Integral solutions.

2010 Mathematics Subject Classification: 11D09

NOTATIONS

Denoting the ranks of some special Polygonal, Gnomonic, Star, Centered Hexagonal and Truncated Tetrahedral numbers.

$T_{3,n} = \frac{n(n+1)}{2}$ = Triangular number of rank n.

$T_{6,n} = n(2n-1)$ = Hexagonal number of rank n.

$T_{8,n} = n(3n-2)$ = Octagonal number of rank n.

$T_{10,n} = n(4n-3)$ = Decagonal number of rank n.

$T_{16,n} = n(7n-6)$ = Hexadecagonal number of rank n.

$T_{18,n} = n(8n-7)$ = Octadecagonal number of rank n.

$Gno_n = 2n-1$ = Gnomonic number of rank n.

$Star_n = 6n(n-1)+1$ = Star number of rank n.

$CH_n = 3n^2-3n+1$ = Centered Hexagonal number of rank n.

$TT_n = \frac{1}{6}\left(23n^2-27n+10\right)$ = Truncated tetrahedral Number of rank n.

$Hendeca_n = \frac{n(9n-7)}{2}$ = Hendecagonal number of Rank n.

$Trideca_n = \frac{n(11n-9)}{2}$ = Tridecagonal number of Rank n.

$Pentadeca_n = \frac{n(13n-11)}{2}$ = Pentadecagonal number of rank n.

$Heptadeca_n = \frac{n(15n-13)}{2}$

= Heptadecagonal number of rank n.
$Nonadeca_n = \frac{n(17n-15)}{2}$ = Nonadecagonal number of Rank n.

I. INTRODUCTION

In Number Theory, Diophantine equations are very often polynomials with integer Coefficients in which the solutions are required to be integers. They have been studied since antiquity and are mathematically both challenging and attractive because of the great diversity of methods that are needed to understand them [1-9]. In particular, refer [10-12] for various problems on binary quadratic equations and in [13-18], Special Pythagorean Problems are studied.

In this communication, we present yet another interesting problem. That is, we search for patterns of Rectangles, where, in each of which, the area is represented by a special polygonal number. Also a few interesting relations among the sides are presented.

II. METHOD OF ANALYSIS

Denoting the Area and Perimeter of the rectangle by A and P respectively.

CASE 1: The assumption

$$A = Hendeca_n$$

leads to the equation

$$2(u+v)(u-v) = n(9n-7)$$

which is satisfied by

$$u = \frac{19n-14}{4}, v = \frac{17n-14}{4}$$

As u and v are to be integers, choose $n = 4\alpha + 14$.

Therefore, $u = 19\alpha + 63, v = 17\alpha + 56$.

Thus, the sides of the Rectangle are given by

$$x = x(\alpha) = 36\alpha + 119$$

$$y = y(\alpha) = 2\alpha + 7$$

The Area and Perimeter of the Rectangle are given by

$$A = A(\alpha) = 72\alpha^2 + 490\alpha + 833.$$

$$P = P(\alpha) = 76\alpha + 252.$$

A few numerical examples are presented below:

TABLE I: AREA AND PERIMETER FOR CASE 1

α	x	y	A	P
1	155	9	1395	328
2	191	11	2101	404
3	227	13	2951	480
4	263	15	3945	556
5	299	17	5083	632

OBSERVATIONS

1. $y(\alpha) \equiv 7 \pmod{2}$.
2. $x + y \equiv 0 \pmod{2}$.
3. $y^2 + T_{6,n} + 6TT_n - 1 \equiv 0 \pmod{29}$.
4. $P(\alpha) \equiv 0 \pmod{4}$.
5. $\alpha x(\alpha) - 119\alpha$ is a perfect square.

CASE 2: The assumption

$$A = Trideca_n$$

leads to the equation

$$2(u+v)(u-v) = n(11n-9)$$

which is satisfied by

$$u = \frac{23n-18}{4}, v = \frac{21n-18}{4}$$

As u and v are to be integers, choose $n = 4\alpha + 18$.

Therefore, $u = 23\alpha + 99, v = 21\alpha + 90$.

Thus, the sides of the Rectangle are given by

$$x = x(\alpha) = 44\alpha + 189. \quad y = y(\alpha) = 2\alpha + 9.$$

The Area and Perimeter of the Rectangle are given by

$$A = A(\alpha) = 88\alpha^2 + 774\alpha + 1701.$$

$$P = P(\alpha) = 92\alpha + 396.$$

A few numerical examples are presented below:

TABLE II: AREA AND PERIMETER FOR CASE 2

α	x	y	A	P
1	233	11	2563	488
2	277	13	3601	580
3	321	15	4815	672
4	365	17	6205	764
5	409	19	7771	856

OBSERVATIONS

1. $x - 21y + T_{8,\alpha} + 2\alpha$ is a Nasty number.
2. $y(\alpha) - Gno_\alpha \equiv 0 \pmod{10}$.
3. $P(\alpha) \equiv 0 \pmod{4}$.
4. $y^2 - T_{10,\alpha} \equiv 0 \pmod{3}$.

CASE 3: The assumption

$$A = Pentadeca_n$$

leads to the equation

$$2(u+v)(u-v)=n(13n-11)$$

which is satisfied by

$$u=\frac{27n-22}{4}, v=\frac{25n-22}{4}$$

As u and v are to be integers, choose $n=4\alpha+22.$.

Therefore, $u=27\alpha+143, v=25\alpha+182.$

Thus, the sides of the Rectangle are given by

$$x=x(\alpha)=52\alpha+275.$$

$$y=y(\alpha)=2\alpha+11.$$

The Area and Perimeter of the Rectangle are given by

$$A=A(\alpha)=104\alpha^2+1122\alpha+3025.$$

$$P=P(\alpha)=108\alpha+572.$$

A few numerical examples are presented below:

TABLE III: AREA AND PERIMETER FOR CASE 3

α	x	y	A	P
1	327	13	4251	680
2	379	15	5685	788
3	431	17	7327	896
4	483	19	9177	1004
5	535	21	11235	1112

OBSERVATIONS

1. $P(\alpha+1)-4\alpha\equiv 0 \pmod 8$.
2. $y(\alpha)\equiv 11 \pmod 2$.
3. $A(\alpha)-13T_{18,\alpha}+2\alpha\equiv 0 \pmod 5$.
4. $2T_{8,\alpha}+2y-22$ is a Nasty number.
5. $(y-10)\times Gno_\alpha$ is a perfect square.

CASE 4:The assumption

$$A=Heptadeca_n$$

leads to the equation

$$2(u+v)(u-v)=n(15n-13)$$

which is satisfied by

$$u=\frac{31n-26}{4}, v=\frac{29n-26}{4}$$

As u and v are to be integers, choose $n=4\alpha+26.$.

Therefore, $u=31\alpha+195, v=29\alpha+182.$

Thus, the sides of the Rectangle are given by

$$x=x(\alpha)=60\alpha+377.\ y=y(\alpha)=2\alpha+13.$$

The Area and Perimeter of the Rectangle are given by

$$A=A(\alpha)=120\alpha^2+1534\alpha+4901.$$

$$P=P(\alpha)=124\alpha+780.$$

A few numerical examples are presented below:

TABLE IV: AREA AND PERIMETER FOR CASE 4

α	x	y	A	P
1	437	15	6555	904
2	497	17	8449	1028
3	557	19	10583	1152
4	617	21	12957	1276
5	677	23	15571	1400

OBSERVATIONS

1. $A(\alpha)-20Star_\alpha-1\equiv 0 \pmod 2$.
2. $30y-x\equiv 0 \pmod{13}$.
3. $10T_{16,\alpha}+\frac{1}{2}P(\alpha)-2\alpha\equiv 0 \pmod{10}$.
4. $(y+13)(y-13)-6\alpha=8Tri_\alpha$.

CASE 5:The assumption

$$A=Nonadeca_n$$

leads to the equation

$$2(u+v)(u-v)=n(17n-15)$$

which is satisfied by

$$u=\frac{35n-30}{4}, v=\frac{33n-30}{4}$$

As u and v are to be integers, choose $n=4\alpha+30.$.

Therefore, $u=35\alpha+255, v=33\alpha+240.$

Thus, the sides of the Rectangle are given by

$$x=x(\alpha)=68\alpha+495.$$

$$y=y(\alpha)=2\alpha+15.$$

The Area and Perimeter of the Rectangle are given by

$$A=A(\alpha)=136\alpha^2+2010\alpha+7425.$$

$$P=P(\alpha)=140\alpha+1020.$$

A few numerical examples are presented below:

TABLE V: AREA AND PERIMETER FOR CASE 5

α	x	y	A	P
1	563	17	9571	1160
2	631	19	11989	1300
3	699	21	14679	1440
4	767	23	17641	1580
5	835	25	20875	1720

OBSERVATIONS

1. $P(\alpha) \equiv 0 (\bmod 10).$
2. $x(\alpha) \equiv 3 (\bmod 4).$
3. $2(x-33y) + Star_{\alpha} + Gno_{\alpha}$ Is a Nasty number.
4. $A(\alpha) - 20T_{15,\alpha} - 2CH_{\alpha} \equiv 1 (\bmod 2).$

III. CONCLUSION

One may search for other patterns of Rectangles under consideration and relations among the solutions.

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