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RESEARCH ARTICLE

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ANALYSIS OF SPLINE SHAFTS UNDER VARIOUS LOADING CONDITIONS USING FINITE ELEMENT ANALYSIS

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ABSTRACT

Mostly failure of rotating machinery occurs due to improper power transmission between the machine elements. Splined connections are widely used in power transmission mechanism. Spline shafts transmit torque from one rotating member to another. This project report presents the study of failure of spline shaft under various stress conditions. The different step size ratio (d/D) is taken as the parameter for finite element analysis.

Combined effect of static torsion and static bending are applied to each model. The maximum Von Mises stress is calculated for the spline shaft with straight sided teeth. The results obtained were compared between the various step size ratio.

In order to reduce the failure of the spline shaft in such cases, it must to study all the stresses accompanying throughout the shaft. For this study, both stepped splined shafts, and partially splined shafts with various step ratios are taken into consideration. The combined effect of static torsion and static bending loads over the spline shafts are considered for analysis.

Keywords: Power transmission, Spline shafts, Von Mises stress, Static bending loads, Failures, Static torsion, Size ratio, Ansys.

1. INTRODUCTION

Spline shafts are the better effective mode of power transmission between to rotating shafts. It employs the use of equally spaced teeth in the radial direction of a cylindrical coordinate system. The teeth on the male spline interlock with the female spline along the longitudinal axis of the cylindrical shaped coupling system. With a substantially larger contact area between teeth when compared to a traditional gear system, spline couplings are used in high torque applications. The load can be distributed in the longitudinal direction of the pressure faces of the teeth.

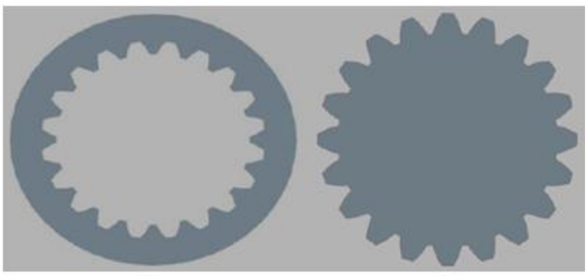


Fig 1.1 Sketch of Spline Shaft and Mating Shaft

Splines have an advantage over shaft key and slot systems. Shafts with external keys are subject to very high stress concentrations at the key root, thus decreasing the fatigue life of the shaft. Shafts with slots cut into them to accept keys are also substantially weakened. Splines allow bending and torsion loads to be distributed over several teeth; each of which acts like a key that is attached to the shaft. This arrangement affords splined connections much greater strength than keyed connections.

Because of their robustness in handling torque, spline couplings are very common to automobiles, ships, gas turbo fan and turbo jet engines in the aerospace industry. A spline coupling is used to transfer torque from the gear book to the sun gear in automobiles. The challenge in spline design, particularly in automobile applications is the effect that the load distribution across the pressure faces of the teeth. Although the pressure faces of the spline teeth provide a large contact area to distribute the torque load; the challenge lies in distributing that load evenly.

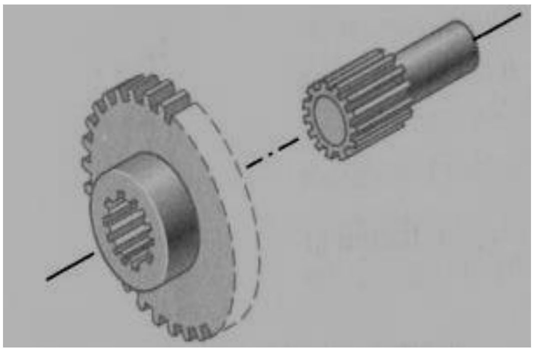


Fig 1.2 Schematic diagram of Spline shaft coupling system

The manufacturing process of a spline has to be done with proper tolerances implemented in it. These tolerances are given in the root fillet radii, the flatness of the pressure faces, and the circular run out of the pitch diameter of the spline coupling. The slight variance of the geometry due to tolerances causes the spline to mate unevenly during torque transfer. The result is that each set of spline teeth sits differently and proper contact is not achieved. These lacks in symmetry causes each tooth to engage slightly with difference in load while each teeth engagement. The tolerances and uneven contact allows the coupling system to vibrate during torque transfer. The vibratory stresses cause the spline teeth wear prematurely from fretting at the pressure faces which leads to failure of the whole spline coupling system. The fretting wear ultimately decreases the life of the part.

2. DESIGN OF SPLINE SHAFT

Splined connections are widely used as a coupling mechanism in rotating machinery. Splined shafts transmit torque from one rotating member to another. The application of the splined connection not only for the transmission of torque alone but also for combined bending and torsional loads. In these cases, design techniques such as tapering or crowning of the spline teeth are used to accommodate such misalignment. When crowned splines are used, the load bearing capacity of the corresponding straight tooth spline is reduced. Thus, a larger diameter shaft must be employed to offset the loss of load carrying capacity.

2.1. Models of Spline shaft and Mating Jaw:

Spline shaft have some advantage over key and slot system. Key is subjected to very high stress which leads to damage of the shaft. The load in spline shafts are equally distributed over all the teeths, which also acts like key. This, equal distribution of load over the shaft affords shaft with spline much greater strength than key and slot connections. A spline shaft consists of two parts; one external spline and another internal spline.Grooves are machine over and into the diameter of shaft to make external and internal spline. The internal and external spline makes the outer and the inner part of connection respectively. The grooves on the internal and external spline shafts should be similar in nature. External splines are manufactured by hobbing, whereas the internal spline is manufactured by sloting as shown in figures 2.1 and 2.2

The splined shaft modeled in this study is used in automobile applications. The shaft transmits torque from a gear box to a planetary gear arrangement. The shaft design incorporates a groove that narrows the shaft cross-section and purposely weakens the shaft. This design feature causes the shaft to fail in rupture. In this way the shaft, which is relatively inexpensive acts as a mechanical before failure of the gear box due to misalignment of the spline shaft and the coupling. Thus, to insure proper failure it is necessary that all stresses be well understood throughout the shaft.

Fig .2.1 Models of Spline shaft and Mating Jaw

Fig 2.2 Power transmission through spline shaft

2.2. Spline Geometry

The general equations used to define the basic proportions of spline teeth are:

$$P = \frac{N}{D} \quad \text{-----------------------(Eq.1)}$$

Where P= the diametral pitch,
N= the number of teeth,
and D= the pitch diameter.

$$p = \frac{\pi D}{N} \quad \text{-------------------------(Eq.2)}$$

Where *p* is the circular pitch.

The main factor that determines engagement is the clearance between mating teeth, commonly referred to as the effective clearance. Some clearance is required to be able to assemble the coupling.

2.3 Tooth Errors

Four main errors affect the effective clearance. They are lead variation error, profile error, tooth thickness error and index variations error. The effective clearance in spline teeth is similar to backlash in gear teeth. The size variations are a result of tooth thickness allowance, which provides clearance for mounting, thermal growth, and lubrication. Tooth errors are caused by local variations in profile, tooth spacing, and tooth thickness as shown in fig2.3.

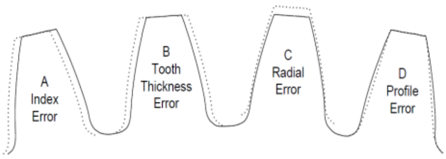


Fig 2.3.Tooth Errors

2.4 Tooth Stresses and Deflections

Spline teeth have been modeled with several different methods. This thesis determines the stress and deflection in teeth. Since splines are essentially gear teeth, the finite element method is employed to determine tooth deflections and stresses for analysis of splines. Spline analysis is simplified because the point of contact does not change as it does in gear teeth. This is because standard spline teeth have short cross-sections and form a very strong beam as shown in fig.2.4.

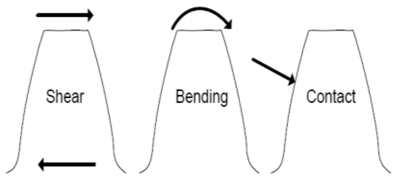


Fig 2.4 Modes of deflections

The teeth analyzed in this research are more similar to gear teeth. However, the stress was not purely due to bending. Other stresses such as hoop stress, torsional stress, and contact stress may contribute to the stress at the tooth fillet as shown in fig2.5.

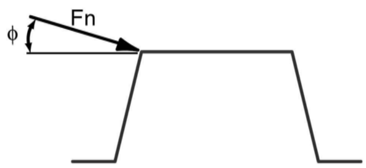


Fig .2.5 Load on the teeth

Because spline teeth are exposed to complex loading, it is important to look at the stress from each component of the reaction force between mating teeth. The reaction force Fn, is represented in figure, which is resolved into three components. Fr is the radial component of the load. Ft is the tangential component, and Mr is the reaction moment due to the eccentric loading of the beam. These are the forces and moments which contribute to the overall stress and deflection of a spline tooth. The following equations resolve Fn into three components, which form an equivalent set of loads:

$$Fr = Fn \cdot \sin\phi$$

$$Ft = Fn \cdot \cos\phi$$

$$Mr = Fr \cdot \frac{t_p}{2}$$

------------------(Eq.3)

Where t_p is the tooth thickness at the pitch circle and f is the pressure angle.

The tangential force Ft, causes a bending moment M, to occur within the beam. The bending moment is greatest at the support, resulting in tensile stress on the loaded side of the beam, and compressive stress on the other. The general equation defining the bending stress in a beam, due to M is

$$\sigma = \frac{Mx}{I}$$

-----------(Eq.4)

Because the radial force acts at the corner of the beam, there is some additional bending stress. This flexural component due to the eccentricity in the loading, is defined by

$$\sigma_{Mr} = \frac{Mr \cdot \frac{t_b}{2}}{I}$$

------------(Eq.5)

The compressive stress due to the radial component of the resultant force is evenly distributed throughout the cross-section of the beam and is defined as

$$\sigma_{Fr} = \frac{Fr}{t_b \cdot l}$$

---------(Eq.6)

Which is the radial force divided by the area at the base of the beam.

2.5 Tooth Engagement

At the point of contact between two mating teeth, there is contact pressure, or a contact stress. The radius of curvature at the point of contact and the load determine how large the contact region is. The internal spline has a negative radius of curvature, while the external spline has a positive radius of curvature. The tooth contact stress is given by

$$P\max = \frac{1}{\pi}\sqrt{\frac{Fn}{l}\frac{\left(\frac{1}{R_i}+\frac{1}{R_e}\right)}{(\zeta_i+\zeta_e)}}$$

----------(Eq.7)

R_i and R_e are the radii of curvature for the internal and the external teeth at the pitch circle.

3. RESULTS

Both the analytical model and the finite element solution use IN-100 powder nickel super alloy for the sleeve and INCO-718 for the shaft when analyzing the coupling system. This is a representative combination for the turbine section of a gas turbo fan engine. The sleeve represents the hub portion of the disk which is closer to the gas path than the shaft which is why IN-100 powder is used. It has more high temperature capability than INCO-718 while still maintaining high strength. A summary of the relevant material properties used in the coupling calculations are summarized in Table 1.

Table 1 – Material Properties of 3D Spline Coupling Model

Specification	Symbol	Sleeve	Shaft	Unit
Material	-	IN-100	INCO 718	-
Density	p	0.284	0.297	lb/in^3
Weight	w	0.118	0.173	lb
Modulus of Elasticity	E	30.1	31.0	Gpa
Shear Modulus	G	11.94	11.10	Gpa
Polar Moment of Inertia	J	0.085	0.037	in^4

Along with material properties, there are two coupling metrics displayed in Table 1. The weight of each component, w, and the polar moment of inertia, J, of each component were calculated by querying the 3D Unigraphics model. The weight and polar moment of inertia can be verified by hand calculations by resolving both geometries into simple shapes and adding the value of each shape in an iterative process.

To compute the axial load distribution on the spline teeth of the shaft, all geometric parameters of the coupling system are defined. Along with geometry the user applied torque is selected as well. Again to match the conditions of a typical gas turbo fan engine, a torque value of 350in-lb is used in calculation of the results. This value is comparable to that transferred between a turbine disk and low pressure shaft. A summary of all geometric parameters and applied torque are displayed in Table 2.

Specification	Symbol	Value	Unit
Applied Torque	τ	350	in-lb
Contact Length	L	0.30	in
Pitch Radius	R	0.70	in
Number of Teeth	N	56	#
Tooth Height	c	0.032	in
Root Fillet Radius	r	0.010	in
Pressure Angle	θ	30	deg
Torsional Stiffness	C_θ	3332488	lb/in-rad

The last parameter listed in table 2 is the torsional stiffness of the coupling system. This value is difficult to determine without the use of the finite element model. Equation 1 defines that the applied torque is equaled to the difference of the angle of twist of the shaft and sleeve multiplied by the torsional stiffness constant, C_θ. Querying the results of the finite element model the difference in the angle of twist between the two components is determined. Plotting the vector sum of deflection for the coupling system, (see Figure 3-1) the maximum and minimum points of deflection of each component are located. The maximum deflection on the shaft is seen at the extreme axial ends of the full hoop section. The minimum deflection of the shaft is seen at the middle of the shaft where the spline teeth are located. This result makes sense intuitively because the outer ends of the shaft should be more flexible than the middle section where the teeth add hoop stiffness.

The sleeve has the opposite response due to the fact the full hoop section is much thicker than the sleeve spline teeth. Therefore, Figure 3-1 maximum defection is seen in the spline teeth and the minimum is seen at the outer edges of the full hoop section.

For each component the difference between the maximum and minimum deflections are determined and resolved into radians based on the pitch radius of the coupling system. Using the relationship that the circumference, s = rϕ, assuming that deflections represent a small arc. Knowing the calculated angle of twist and the applied torque the torsional stiffness constant is estimated as shown in table 2.

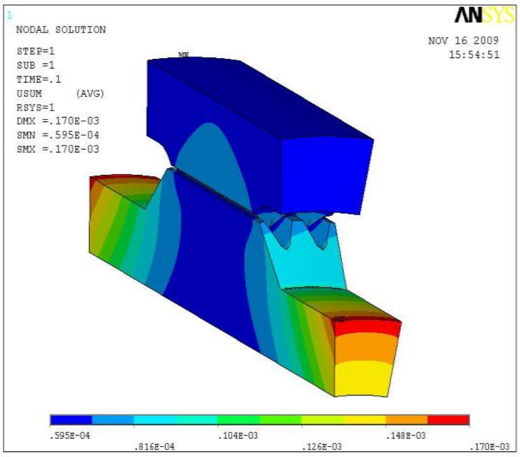


Figure 3-1: 3D vector sum of deflection in the spline coupling from applied torque

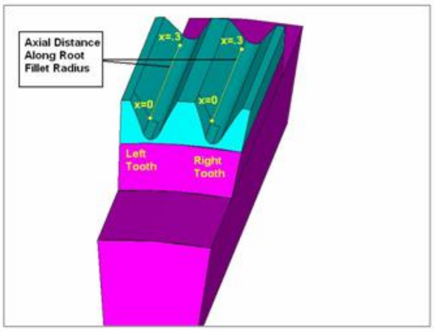
With all of the coupling specifications defined, the values can be substituted into equations: 12, 13, 14, 17 and 18. These equations represent the analytical solution of load distribution at the root fillet radius of the spline teeth of the shaft. The calculated values of the equations are shown in table 3.

The analytical response is compared to the finite element solution. The axial load distribution at the root fillet radius of the spline teeth of the shaft is extracted from the finite element model. See Figure 3-2, which shows the path used to extract the load seen at the root fillet radius of both teeth. The path distance is equaled to the contact length of the spline teeth, L = x = 0.3 inches.

Table 3 – Analytical results for axial load distribution at root fillet radius

Parameter	Value	Unit
α	10.67	$(lb/in\text{-}rad)^{1/2}$
A	13.67	-
B	336.3	-
$p(x)_{max}$	97.66	ksi
p_{avg}	57.61	ksi
PR	1.70	-

Figure 3-2: Schematic showing the path used to extract the load distribution in the finite element model



The extracted load distribution along the contact length of the spline teeth is summarized in table 4 for comparison to the analytical model summarized in table 3.

Table 4 – Finite element results for axial load distribution at root fillet radius

Parameter	Left Tooth	Right Tooth	Unit
$p(x)_{max}$	71.13	68.51	ksi
p_{avg}	47.06	45.34	ksi
PR	1.51	1.51	-
$d(x)_{max}$	0.00015	0.00015	in
d_{avg}	0.0014	0.00014	in
DR	1.1	1.1	-

The parameters d(x), d_{avg}, and DR represent the deflection of the spline teeth as a function of x, the average deflection over the contact length, and the deflection ratio respectively. The deflection ratio, DR, is the ratio of the maximum deflection, d(x) over the average deflection seen across the root fillet radius of the shaft spline teeth. Deflection is book kept in the finite element model to determine whether axial deflection in the spline teeth influences the load distribution. However, table 4 shows that the deflection across the root fillet radii of the spline teeth is uniform, with a DR of 1.1.

The finite element solution of load distribution along the plotted path in figure 3-2 is not uniform. The load peaks at either end of the contact length as displayed by the finite element response of torque transfer presented in figure 3-3.

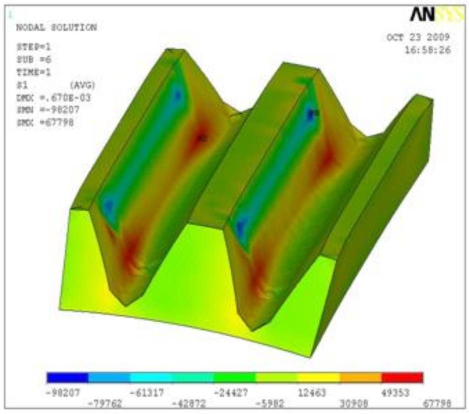


Figure 3-3: Finite element model axial load distribution in the shaft spline teeth due to the applied torque

When observing the entire pressure face of the spline teeth, the red color shows that the highest stress is found in the root fillet radius along the contact length. It also shows that the load in the fillet is not distributed evenly. This result is consistent with the plot of the analytical solution (see Figure 3-4).

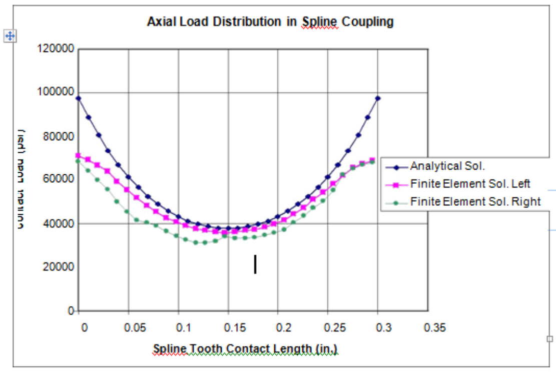


Figure 3-4: Plot of contact pressure at root fillet radius verses contact length

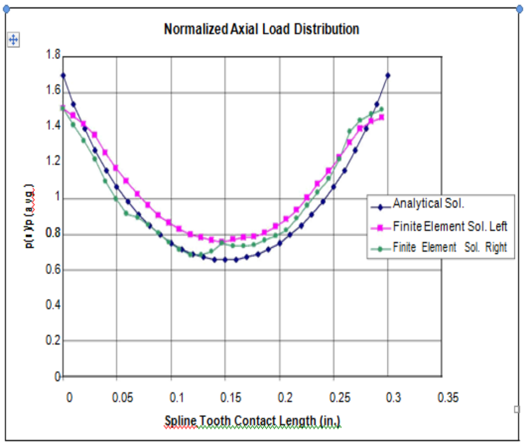


Figure 3-5: Plot of normalized contact pressure, p(x)/p(avg), at the root fillet radius of the spline teeth verses contact length

Finite element solution produce the same shape of load distribution, the analytical model predicts higher peaks. The maximum analytical load is 97.6 ksi, while the finite element equivalent is 68.2, which leaves a 30 ksi disparity between the two. Furthermore, referring to tables 3 and 4, the pressure ratio of the analytical model is 1.70 while the finite element pressure ratio is 1.51.

The disparity between the analytical and finite element solutions can be explained by the two facts. First, is that the analytical solution has with it inherent assumptions discussed in the methodology. It assumes one hundred percent torque transfer from the sleeve to the shaft, which means the analytical load distribution is a result of zero loss of energy to friction or deformation. In actuality, as the finite element solution shows, there is not one hundred percent torque transfer. A portion of the torque goes into deforming the spline teeth. Looking at the pressure faces of the teeth in Figure 3-3, the blue and green represents the portion of the tooth under compression, while the red and yellow represent the portion under tension. Each shaft tooth is acting as a cantilever beam where the torque transfer from the sleeve supplies the load at the end of each shaft tooth. Because some of the torque causes the teeth to bend, the root fillet radius does not see the entire load. That is why, in Figure 3-4, and Figure 3-5, the analytical solution has higher peaks.

The second reason for the disparity in load distribution and pressure ratio between the analytical and finite element solution is the fact that the analytical does not take into account the distorted shape during torque transfer. The 3D finite element model has two pairs of deformed teeth that mesh with each other (see Figure 3-6, and Figure 3-7). Because the pressure faces of the teeth do not meet flush during torque transfer, the root fillet radius does not see the entire load. This explains why the analytical solution has a higher p_{avg} and p_{max} which results in a 12.5% higher pressure ratio.

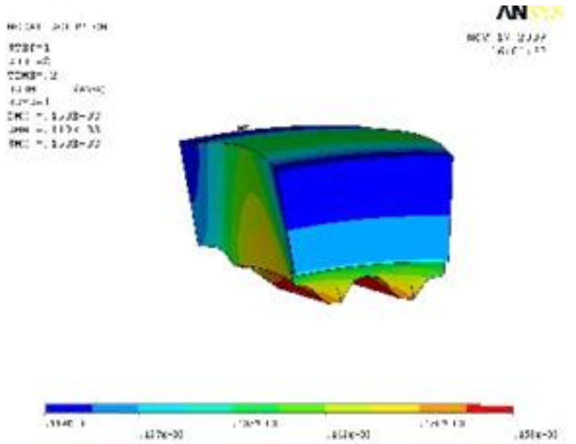


Figure 3-6: 10X distortion of the 3D deflection of the sleeve spline teeth under torque load

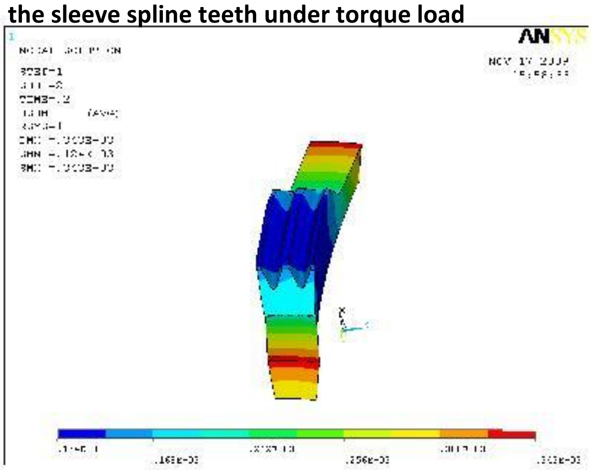


Figure 3-7: 10X distortion of the 3D deflection of the shaft spline teeth under toque load

Figure 3-4 and figure 3-5 show the disparity between the analytical solution and the finite element solution although both cases exhibit the same shaped curve of load distribution. The normalized load distribution plot causes both cases to more closely align, which is a sign that the analytical and finite element solutions agree when the assumptions are factored out. Also, it is important to note that the finite element model shows consistent behavior when comparing the right tooth to the left tooth. This symmetry points to the fact that the boundary conditions are properly applied and match those used in the analytical methodology.

Figure 3-3, figure 3-6 and figure 3-7 help to explain why the finite element model predicts a lower load distribution and lower pressure ratio. The 3D effects of the tooth bending and the fact that the pressure faces of the teeth do not exhibit a flush mating surface because of torsional distortion, cause an imperfect torque transfer system. From the current analysis this loss of torque can be quantified to approximately 25%. The 3D effects reduce the load seen at the root fillet radius. However, these effects simulate the loading condition a designer can expect during operation in an engine.

When comparing the finite element to the analytical both have advantages and disadvantages. The finite element solution gives an accurate analysis of the coupling design. The finite element model predicts a lower PR than the analytical equation. With all else being equal, by correlation, the finite element model proves that there is approximately 12.5% more life in the coupling system. Although the finite element model is more realistic it is a much lengthier method of determining the effectiveness of the spline design. The analytical equation is conservative but it gives an instant answer when the geometric and material properties are plugged into equations 12-18. The designer will have to weigh his or her options on whether or not 12.5% more predicted life is value added when compared to the time savings of a shorter analysis.

4. CONCLUSION

The finite element model incorporates the spline coupling material properties, geometry, and boundary conditions into a 3D analysis. Symmetry and accuracy of the boundary conditions in the finite element model are justified by the fact that both the left and right tooth exhibit the same response to the applied torque load. The 3D finite element solution shows that the 25% of the torque is dissipated during transfer. A portion of that coupled force goes into bending the spline teeth and twisting the overall spline geometry. The 3D finite element model is more accurate, and the stress developed on the right tooth has much more stress when compared to that of the left tooth. This increases the wear rate of the coupling compared to that of spline shaft which leads to failure when sudden torsional load is applied.

REFERENCES

[1]. Paul E. Burke, Walter Fisher, *Design and Analysis Procedures for Shaft and Splines*, SAE International, 2008

[2]. Kin S. Yeung, *Analysis of a new concept in spline design for transmission output shafts,* 2007

[3]. Robert A Adey, Colin McFarlane, James W Taylor, *Modeling and Simulation of Spline Couplings*, 2006

[4]. Sum, W. S., 2005. "*Efficient finite element modeling for complex shaft couplings under non-symmetric loading*," Journal of Strain Analysis for Engineering Design

[5]. Barrot, A., Paredes, M., and Sartor, M., 2008, "*Extended equations of load distribution in the axial direction in a spline coupling*," Engineering Failure Analysis

[6]. Ding, J., McColl, I. R., Leen, S. B., 2006. "*The application of fretting wear modeling to a spline coupling*

[7]. K J Bathe, *Finite Element Procedures*,