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# APPLICATION OF PSO AND ITS VARIENTS TO BILATERAL AND MULTILATERAL TRANSACTIONS IN DEREGULATED POWER SYSTEM

**M.VEERA KUMARI[1], B.KALYANI[2]**
[1]Department of Electrical & Electronics Engineering, SIR C R Reddy College of Engineering, Eluru
[2]Department of Electrical & Electronics Engineering, SIR C R Reddy College of Engineering, Eluru

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## ABSTRACT

Deregulation of the electric power industry is one of the important aspects in power sector. The Deregulation of the electric power industry is intended to create market conditions, competitions and innovation etc. Under deregulation, among various issues, managing dispatch is an important control activity in a power system. The control variable has to be obtained by optimizing different objectives of choice by satisfying the power flow. Modern electricity markets offer the possibility of ex-changing power among market participants in different ways. All generated power was centrally dispatched from generators to loads, nowadays unbundling and open transmission access gives the possibility of having direct agreement between generators and loads. Moreover, this brisk and lucrative market enables other purely financial players to participate in the games. Thus, individual generators can sell power directly to loads, to a pool, or to trading entities, leading transactions to assume a double aspect: both physical and financial. Typical OPF solution adjusting the appropriate control variables, so that a specific objective in operating a power system network is optimized (maximizing or minimizing) with respect to the power system constraints. The OPF is also suited for deregulated environment and can solve some contractual dispatch, i.e. Bilateral and Multilateral dispatch. In this thesis, bilateral transactions and multilateral transactions, which are likely to occur in deregulated energy markets, are simulated. The Particle Swarm Optimization (PSO) along with its variants mainly TPSO are applied to obtain optimal power flow problem with bilateral and multilateral transactions. The performance is studied on IEEE 9 bus, IEEE 14 bus system for minimization of active power loss and minimization of loss by considering real power generation and bus voltages as control variables.

Key Words – PSO, power flow, optimization ,OPF, TPSO

## INTRODUCTION

The Optimal Power Flow (OPF) has been widely used for both the operation and planning of a power system. Therefore, a typical OPF solution adjusting the appropriate control variables, so that a specific objective in operating a power system network is optimized (maximizing or minimizing) with respect to the power system constraints, dictated by the electrical network. The OPF is also suited for deregulated environment and can solve some contractual dispatch, i.e. Bilateral and Multilateral dispatch.

Deregulation basically means that the generation portion of electricity service will be open to competition. However, the transmission and distribution of the electricity service will remain regulated. A bilateral transaction between a supplier and a buyer involves the injection of power at one location in the network and the extraction of the same amount of power, at the same time, at another location. Multilateral transactions are an extension of bilateral transactions. In a multilateral transaction, there are many generation points (at least more than one), similarly there are many load points (at least more than one). T Particle Swarm Optimization (PSO) is an evolutionary algorithm that may be used to find optimal (or near optimal) solutions to numerical and qualitative problems.

Turbulence Particle Swarm Optimization (TPSO) uses a minimum velocity threshold to control the velocity of the particles. TPSO mechanism is similar to a turbulence pump, which supplies some power to the swarm system to explore new neighbourhoods for better solutions . The algorithm also avoids clustering of particles and at the same time attempts to maintain diversity of population.

## OPTIMAL POWER FLOW

Optimal power flow (OPF) has been widely used in power system operation and planning. In deregulated environment of power sector, it is of increasing importance, for determination of electricity prices and also for congestion management.OPF is a computationally intensive tool when analysing many generation plants, transmission lines and demands. Finally the engineering constraints and economic objectives for system operations are combined by formulating and solving the optimal power flow problem. OPF is used in economic analysis of the power system as well.

Optimal Power Flow (OPF) is a method to find steady state operation point which minimizes generation cost , loss etc. or maximizes social welfare, load ability etc while maintaining an acceptable system performance in terms of limits on generator's real and reactive powers, line flow limits, output of various compensating devices etc. The OPF problem may also have the formulation of active power generation dispatch (Economic Dispatch Problem, EDP) and reactive power generation dispatch .The main purpose of the EDP is to determine the generation schedule of the electrical energy system that minimizes the total generation and operation cost and does not violate any of the system operating constraints such as line overloading, bus voltage profiles and deviations.

***General Opf Formulations:*** In general, the mathematical formulation of the OPF problem can be formulated as Constrained non- linear optimization problem discussed below:

*Minimize:*

$$f(x,u) \ldots\ldots(1)$$

*Subject to*:

$$f_E(x,u) = 0 \ldots\ldots(2)$$

$$f_0(x,u) \le 0 \ldots\ldots(3)$$

$$f_c(x,u) \le 0 \ldots\ldots.(4)$$

The objective function is a scalar function. Two types of variables appear in the above

***Optimization problem:*** *x* is a set of state variables (voltage magnitudes *v* and phase angles θ for each node in the Network) and *u* is the set of controllable quantities in the system (generator outputs, adjustable transformers)

$$x = (\frac{v}{\theta}) \ldots\ldots\ldots.(5)$$

Also,

$$u = (\cfrac{P_g}{\cfrac{Q_g}{\cfrac{t_b}{\phi}}}) \ldots\ldots\ldots..(6)$$

Where, The number of control variables are active power ( $P_g$)

Reactive power ( $Q_g$ ),

Tap Changing transformers ($t_b$),

Phase shifting transformers (φ).

***(a)The Objectives***

*(i)Minimization of Generation Fuel Cost:*

The objective function is the minimization of the generation fuel cost. Generally, the OPF generation fuel cost function can be expressed by a quadratic function as follows:

Minimize

$$f_T=\sum_{i=1}^{Ng} f_i(P_{gi})........(7)$$

$$f_i(P_{gi})=a_i+b_iP_{gi}+c_iP_{gi}^2.......(8)$$

Where:

$N_g$ is the number of generators including the slack generator in any electric network.

$a_i$ is the basic cost coefficient of the $i^{th}$ generator

$b_i$ is the linear cost coefficient of the $i^{th}$ generator

$c_i$ is the quadratic cost coefficient of the $i^{th}$ generator

$p_{gi}$ is the real power output of the $i^{th}$ generator $p_g$ is the vector of real power outputs of all generator units and is defined as

$$Pg=[P_{g1},P_{g2},............P_{gn}]^T.....(9)$$

*(ii)Minimization of Active Power Transmission Loss:*

Active power loss plays a great role in solving OPF problem. It can be obtained by Subtracting active power (generation) from active power (demand). The expression for active power loss is as below :

$$P_L=\sum P_{gi}-\sum P_{di}.....(10)$$

The term $p_i$ in the above two equations represents the total $I^2R$ loss in the transmission lines and transformers of the network

***(b)The Constraints***

*(i)Equality Constraints:* The equality constraints of the OPF reflect that the net injection of the real and reactive power at each bus to be zero as shown:-

The power flow equation of the network

$$f(V,\phi)=0........(11)$$

Where

$$f(V,\phi)=P_i(V,\phi)-P_i^{net}........(12)$$

$$Q_i(V,\phi)-Q_i^{net}.........(13)$$

$$P_m(V,\phi)-P_m^{net}.........(14)$$

Where:

- Pi and Qi are respectively calculated real and reactive power for PQ bus i. $p_i^{net}$ and $q_i^{net}$ are respectively calculated real and reactive power for PQ bus i.
- $P_m$ and $P_{mi}$ are respectively calculated and specified real power for PV bus m.
- V and f are voltage magnitude and phase angles at different buses.

*(ii)Inequality constraints:*The inequality constraint on reactive power generation $Q_{gi}$ at each PV bus

$$Q_{gi}^{\min}\le Q_{gi}\le Q_{gi}^{\max}......(15)$$

Where $Q_{gi}^{\min}$ and $Q_{gi}^{\max}$ are respectively minimum and maximum value of reactive Power at PV bus i.

The inequality constraint on phase angle $\Phi_i$ of voltage at all buses i.

$$\phi_i^{\min}\le\phi_i\le\phi_i^{\max}.........(16)$$

Where $\phi_i^{\min}$ And $\phi_i^{\max}$ are respectively minimum and maximum phase angle at bus i.

MVA flow limit on transmission line

$$MVAjj\le MVA_{jj}^{\max}....(17)$$

Where

$MVAjj$ and $MVA_{jj}^{\max}$ is the maximum rating of transmission line connecting bus i and j

***The Control Variables***

The control variables are described as below:

Only Pg as a control variable

$$[P_1,P_2.................................P_n].....(18)$$

Pg and generator bus voltages as control variable

$$[P_1,P_2.................................P_n,V_1,V_2................V_n].......(19)$$

**POWER SYSTEM DEREGULATION**

The electricity market has experienced enormous setbacks in delivering on the promise of deregulation. In theory, deregulating the electricity market would increase the efficiency of the industry by producing electricity at lower costs and passing those cost savings on to customers. For the electric industry, deregulation means the generation portion of electricity service will be open to competition. However, the transmission and distribution of the electricity will remain regulated and our local utility company will continue to distribute electricity to us and provide customer services to us. The generation of electricity is being deregulated, which means we will have the opportunity to shop around for the electricity-Generation supplier of choice. India also had a centralized institutional environment for the provision of electricity. India's power generation management in India was structured by the Electricity (Supply) Act of 1948. The 1948 Act provided for the establishment of a Central Electricity Authority (CEA). The CEA was established with the purpose of developing a uniform national power policy and of providing clearance for power projects.

The 1948 Act also set up a network of state electricity boards (SEBs), power generation undertakings and management boards under central or joint partnership for the purpose of meeting regional power requirements. Currently, there are 18 SEBs that generate about two-thirds of the country's transmission, distribution and supply of electricity.

***(i) Bilateral transactions:*** Bilateral transactions are contracts between power sellers (say GENCOS) and buyers (say DISCOS or large customers). This type of power trading would therefore entail injection of bulk power into the transmission system by the power producer, and withdrawal of an equal amount of power, from the network by the customer. This scenario is simulated as the output of a generator, at a generator bus, and a corresponding load, at a load bus. Following assumptions are resorted to:

- there are multiple candidate power suppliers, being included in the generation set;
- there are multiple candidate power customers, being included in the load set;
- the reactive power of load is compensated locally, only real power transaction is supplied by the transaction;

The transmission system has enough transmission capability to carry this amount of MW transaction sent from a supplier to a customer. This alternatively implies, that the transaction amount is decided after taking into account the results of the Available Transfer Capability (ATC) analysis.

***(ii) Multilateral transactions:*** Multilateral transactions are an extension of bilateral transactions. It is a trade that is arranged by energy brokers. In a multilateral transaction, there are many generation points (at least more than one), similarly there are many load points (at least more than one). The scheduling coordinator (SC) of a group of multilateral transactions provides the maximum as well as proposed generation and demand at different generation and demand points, respectively. The coordinator also provides the maximum and proposed demands at different load points of the group. The SC determines the feasibility of this group of multilateral transaction and suggests minimum possible curtailments . After finalization, the feasible multilateral transaction is scheduled.In the case of multi-lateral transaction, the summation of power injected in different buses *(i)* is equal to the summation of load powers taken out at various buses *(j)*.

$$\sum_i P_{gi}^k - \sum_j P_{dj}^k = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.21)$$

Where

K= 1, 2, 3…….$t_k$.

Where $P_{gi}$ and $P_{dj}$ represent the power injection into the seller bus- *i* and the power taken out at buyer bus- *j*, $t_k$ is the total number of transactions.

**TURBULENT SWARM OPTIMIZATION**

A new velocity updates approach for the particles in PSO and analyzes its effect on the particle's behavior. We also illustrate a Fuzzy Logic Controller (FLC) scheme to adaptively control the parameters (Herrera and Lozano, 2003; Mark and Shay, 2005; Yun and Gen, 2003). One of the main reasons for premature convergence of PSO is due to the stagnation of the particles exploration of a new search space. We introduce a strategy to drive those lazy particles and let them explore better solutions. If a particle's velocity decreases to a threshold *vc*, a new velocity is assigned using the below equation. Thus, we present the TPSO using a new velocity update equations:

$$v_{ij}(t) = w\hat{v} + c_1 r_1 (x_{ij}^{\#}(t-1) - x_{ij}(t-1)) + c_2 r_2 (x_j^{*}(t-1) - x_{ij}(t-1)) \ldots\ldots (20)$$

$$\hat{v} = \begin{cases} v_{ij}\, if \left|v_{ij}\right| \geq v_c \\ u(-1,1) v_{\max/\rho}\, if \left|v_{ij}\right| < v_c \end{cases} \ldots\ldots (21)$$

Where *u (*−1*,* 1*)* is the random number, uniformly distributed with the interval [−1, 1] and *ρ* is the scaling factor to control the domain of the particle's oscillation according to $v_{max}$. $v_c$ is the minimum velocity threshold, a tunable threshold parameter to limit the minimum of the particles' velocity. The change of the particle's situation is directly correlated to two parameter values, $v_c$ and *ρ*. A large $v_c$ shortens the oscillation period and it provides a great probability for the particles to leap over local minima using the same number of iterations. But a large $v_c$ compels particles in the quick 'flying' state, which leads them not to search the solution and forcing them not to refine the search. In other words, a large $v_c$ facilitates a global search while a smaller value facilitates a local search.

By changing it dynamically, the search ability is dynamically adjusted. The value of *ρ* changes directly the particle oscillation domain. It is possible for particles not to jump over the local minima if there would be a large local minimum available in the objective search space. But the particle trajectory would more prone to oscillate because of a smaller value of $\rho$. For the desired exploration–exploitation trade-off, we divide the particle search into three stages. In the first stage the values for *vc* and $\rho$ are set at large and small values, respectively. In the second stage, *vc* and $\rho$ are set at medium values and in the last stage, *vc* is set at.

TPSO as an alternative method to overcome the problem of premature convergence in the conventional PSO algorithm. TPSO uses a minimum velocity threshold to control the velocity of particles. TPSO mechanism is similar to a turbulence pump, which supply some power to the swarm system. The basic idea is to control the velocity the particles to get out of possible local optima and continue exploring optimal search spaces. The minimum velocity threshold can make the particle continue moving and maintain the diversity of the population until the algorithm converges.

In the paper the variants worked out are constriction factor based particle swarm optimization approach (CFBPSO) and Turbulent Particle Swarm optimization technique (TPSO).

**SIMULATION RESULTS**

The simulation has been carried out on system having MATLAB R2010a using MATPOWER 4.1 .The specifications for the pc are of dell Intel core processor with i5 configuration and results are viewed taking active power loss as objective function and reactive power loss as objective function with branch number on real axis for the studies The OPF using PSO has been carried out on the IEEE 14 and IEEE 9 bus system The data considered for the IEEE 14, 9 bus systems are given in Appendix. The OPF solution has been attempted for minimizing the generation cost, active and reactive power loss by considering the (i) Generation Pg's, (ii) Generation Pg's and generator bus voltages as control variables.

**RESULTS FOR 9 BUS SYSTEMS WITH OUT PSO**

Table 1 : shows the comparison of real power flows without pso

| From –To Branches | BASE CASE P(MW) Power flow From-To | | CASEI P(MW) Power flow From-To | | CASEII P (MW) Power flow From-To | | CASE III P(MW) Power flow From-To | |
|---|---|---|---|---|---|---|---|---|
| 1-4 | 89.80 | -89.80 | -115.72 | 115.72 | 96.31 | -96.31 | 98.03 | -98.03 |
| 4-5 | 35.22 | -35.04 | -0.0001 | 0.0001 | 42.32 | -42.06 | 41.38 | -41.14 |
| 5-6 | -54.96 | 55.97 | 00.47 | 0.13 | -57.94 | 59.07 | -58.86 | 60.03 |
| 3-6 | 94.19 | -94.19 | 50.206 | -56.206 | 99.88 | -99.88 | 101.28 | -101.28 |
| 6-7 | 38.22 | -38.07 | 0.0035 | -43.00 | 40.81 | -40.64 | 41.25 | -41.08 |
| 7-8 | -61.93 | 62.21 | 0.0004 | -0.20 | -69.36 | 69.71 | -68.92 | 69.27 |
| 8-2 | -134.32 | 134.32 | 0.3781 | -37.81 | -142.43 | 142.43 | -144.45 | 144.45 |
| 8-9 | 72.11 | -70.72 | 0.10 | -0.0001 | 72.71 | -71.3 | 75.19 | -73.67 |
| 9-4 | -54.28 | 54.58 | -0.40 | 0.30 | -53.7 | 53.99 | -56.33 | 56.64 |

**RESULTS FOR 9 BUS SYSTEM WITH PSO**

Table 2 : shows the comparision of real power flows with pso

| From –To Branches | BASE CASE P(MW) Power flow From-To | | CASEI P(MW) Power flow From-To | | CASEII P (MW) Power flow From-To | | CASE III P(MW) Power flow From-To | |
|---|---|---|---|---|---|---|---|---|
| 1-4 | 89.80 | -89.80 | 93.22 | -93.22 | 96.31 | -96.31 | 98.03 | -98.03 |
| 4-5 | 35.22 | -35.04 | 40.86 | -40.62 | 42.32 | -42.06 | 41.38 | -41.14 |
| 5-6 | -54.96 | 55.97 | -59.38 | 60.56 | -57.94 | 59.07 | -58.86 | 60.03 |
| 3-6 | 94.19 | -94.19 | 97.04 | -97.04 | 99.88 | -99.88 | 101.28 | -101.28 |
| 6-7 | 38.22 | -38.07 | 36.48 | -36.34 | 40.81 | -40.64 | 41.25 | -41.08 |
| 7-8 | -61.93 | 62.21 | -63.66 | 63.95 | -69.36 | 69.71 | -68.92 | 69.27 |
| 8-2 | -134.32 | 134.32 | -138.35 | 138.35 | -142.43 | 142.43 | -144.45 | 144.45 |
| 8-9 | 72.11 | -70.72 | 74.4 | -72.92 | 72.71 | -71.3 | 75.19 | -73.67 |
| 9-4 | -54.28 | 54.58 | -52.08 | 52.36 | -53.7 | 53.99 | -56.33 | 56.64 |

## RESULTS FOR 9 BUS SYSTEM WITH TPSO

Table 3 : shows the comparison of real power flows with Tpso

| From –To Branches | BASE CASE P(MW) Power flow From-To | | CASE I P(MW) Power flow From-To | | CASE II P (MW) Power flow From-To | | CASE III P(MW) Power flow From-To | |
|---|---|---|---|---|---|---|---|---|
| 1-4 | 89.80 | -89.80 | 93.02 | -93.02 | 96.21 | -96.21 | 97.88 | -97.88 |
| 4-5 | 35.22 | -35.04 | 40.66 | -40.42 | 42.22 | -41.96 | 41.23 | -40.99 |
| 5-6 | -54.96 | 55.97 | -59.18 | 60.36 | -57.84 | 58.93 | -58.71 | 59.88 |
| 3-6 | 94.19 | -94.19 | 96.84 | -96.84 | 99.78 | -99.78 | 101.13 | -101.13 |
| 6-7 | 38.22 | -38.07 | 36.28 | -36.14 | 40.71 | -40.54 | 41.1 | -40.93 |
| 7-8 | -61.93 | 62.21 | -63.46 | 63.75 | -69.26 | 69.61 | -68.87 | 69.12 |
| 8-2 | -134.32 | 134.32 | -138.13 | 138.15 | -142.33 | 142.32 | -144.3 | 144.3 |
| 8-9 | 72.11 | -70.72 | 74.2 | -72.72 | 72.61 | -71.2 | 75.04 | -73.52 |
| 9-4 | -54.28 | 54.58 | -51.88 | 52.16 | -53.6 | 53.89 | -56.18 | 56.49 |

The simulation has been carried out on system having MATLAB R2010a using MATPOWER 4.1 .The specifications for the pc are of dell Intel core processor with i5 configuration and results are viewed taking active power loss as objective function and reactive power loss as objective function with branch number on real axis for the studies.

The various transactions for this system are carried out as below:

- Base case where no power transactions has been carried out For IEEE 14 bus system
- CASE I when 10 MW power injected at generator bus 5 and drawn at load bus 2
- CASE II when 20 MW injected at the generator bus 5 and drawn at load buses 2 and 4
- CASE III when 25 MW is injected at generator bus 5 and drawn at the load buses 2,4, 6 as 10MW , 10 MW, and 5MW respectively.

The optimal power flow solution with particle swarm optimization is done on different cases and the real power injections, reactive power injections are observed in bilateral and multilateral transactions. As a case study an IEEE 9 bus system is also considered.In this system similar to 14 bus system different transactions are analyzed bilalateral and multilateral methods are implemented.

Different cases studied in 9 bus system are :

(1) Base case where no power transactions take place which acts as reference for other cases
(2) Case I : When 10 MW power injected at the generator bus 3 and drawn at the load bus 5
(3) Case II : When 20 MW power injected at the generator bus 3 and drawn at the load buses 5 and 7 each of 10 MW respectively
(4) Case III : when 5 MW , 20 MW are injected at generator buses 2 , 3 respectively and drawn at the load buses 5,7,9 each of 10 MW,10 MW, 5 MW correspondingly.

Table 4 : shows the comparison of losses in different cases

| S.NO | BASE CASE | CASE I | CASE II | CASE III |
|---|---|---|---|---|
| WITH OUT PSO | 3.307 | 3.615 | 3.610 | 3.760 |
| WITH PSO | 2.0260 | 2.7952 | 3.3467 | 2.6715 |
| WITH TPSO | 2.0001 | 2.7722 | 3.2203 | 2.4205 |

## CONCLUSION

In this paper Bilateral and multilateral power flows under deregulated Power system environment are studied. The proposed model used to solve the optimal power flow in deregulated environment. The performance of the IEEE 14-bus and IEEE 9 bus test system for fuel cost minimization, minimization of active power loss by considering real power generation and bus voltages as control variables. The Particle Swarm Optimization (PSO) along with its variants such as , Turbulent swarm optimization (TPSO) is applied to obtain optimal power flow problem with bilateral and multilateral transactions. Generators with input/output cost characteristic curves such as exponential cost curve, quadratic curve are used. The algorithm has accurately and reliably converged to the global optimum solution in each case.

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