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RESEARCH ARTICLE

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# ON THE HYPERBOLA

$$ax^2-(a-1)y^2=a\,,a>1$$

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**ABSTRACT**

The binary quadratic equation representing hyperbola given $ax^2-(a-1)y^2=a\,,a>1$ is analysed for determining its non- zero distinct integer points. The recurrence relations satisfied by x and y are given. A few interesting relations among the solutions are presented.

**KEYWORDS:** Binary quadratic , integer points, hyperbola

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**INTRODUCTION:**

The binary quadratic Diophantine equations offer an unlimited field for research because of their variety [1-2]. For an extensive review of various problems one may refer [3-25] . This communication concerns with yet another interesting binary quadratic equation $ax^2-(a-1)y^2=a\,,a>1$ representing hyperbola for determining its infinitely many non-zero integral solutions . A few interesting properties among the solutions are presented.

**METHOD OF ANALYSIS :**

The hyperbola under consideration is

$$ax^2-(a-1)y^2=a\,,a>1 \quad (1)$$

Introduction the linear transformations

$$x=X+(a-1)T \text{ and } y=X+aT$$

in (1), it is written as

$$X^2=T^2(a^2-a)+1$$

which is a pellian equation , whose general solution $(\tilde{X}_n,\tilde{T}_n)$ is given by

$$\tilde{X}_n=\frac{1}{2}f_n$$

$$\tilde{T}_n=\frac{1}{2\sqrt{a^2-a}}g_n$$

where

$$f_n=\left((2a-1)+2\sqrt{a^2-a}\right)^{n+1}+\left((2a-1)-2\sqrt{a^2-a}\right)^{n+1}$$

$$g_n=\left((2a-1)+2\sqrt{a^2-a}\right)^{n+1}-\left((2a-1)-2\sqrt{a^2-a}\right)^{n+1}$$

In view of (2), the integer values of $x$ and $y$ satisfying (1) are given by

$$x_{n+1}=\frac{2a-1}{2}f_n+\sqrt{a^2-a}\,g_n$$

$$y_{n+1}=af_n+\frac{\sqrt{a^2-a}}{2}\left(\frac{2a-1}{a-1}\right)g_n \quad (2)$$

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The recurrence relations satisfied by the $x$ and $y$ values of (1) are given by

$$x_{n+3} - (4a-2)x_{n+2} + x_{n+1} = 0$$

$$x_0 = 2a-1 \text{ and } x_1 = 8a^2 - 8a + 1$$

Similarly,

$$y_{n+3} - (4a-2)y_{n+2} + y_{n+1} = 0$$

, $y_0 = 2a \text{ and } y_1 = 8a^2 - 4a$

A few interesting relations among the solutions are as follows:

(1). $x_{n+2} = (a-1)y_{n+1} - x_{n+1}$

(2). $x_{n+3} = (64a^4 - 128a^3 + 92a^2 - 2$
$160a^3 + 124a^2 - 26a -$

(3). $y_{n+1} = \frac{1}{a-1}(x_{n+2} + x_{n+1})$.

(4). $y_{n+2} = \left(\frac{a^2-3a+1}{a-1}\right) y_{n+1} - \left(\frac{a^2+2a}{a-1}\right) x_{n+1}$

.

(5). $y_{n+3} = \left(\frac{8a^3-16a^2+8a-1}{a-1}\right) y_{n+1} - \left(\frac{76a^3+12a^2-4a}{a-1}\right) x_{n+1}$

(6). $(4a-2)\, x_{2n+2} - (4a-4)y_{2n+2} + 2$ is a perfect square.

(7). $6((4a-2)x_{2n+2} - (4a-4)y_{2n+2} + 2)$ is a nasty number

(8). $(4a-2)x_{3n+3} - (4a-4)y_{3n+3} + 3f_n$ is a cubical number

**REMARKABLE OBSERVATION:**

Let

$$\alpha_{n+1} = (4a-2)x_{n+1} - (4a-4)y_{n+1}$$

$$\beta_{n+1} = (4a^2 - 6a + 2)y_{n+1} - 4(a^2 - a)x_{n+1}$$

Note that the pair $(\alpha_{n+1}, \beta_{n+1})$ satisfies the hyperbola

$$\beta_{n+1}^2 = (a^2 - a)\alpha_{n+1}^2 + 4(a - a^2)$$

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